UNIVERSITÄT BONN
HRL
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HUMANOID ROBOTS LAB
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Humanoid Robotics
Submission until 09.07.2026 (10:00).
Discussion in Tutorial on 14.07.2026.
Important: Solution should be **type-written** and handed in as pdf, handwritten solutions are **not** accepted. The solution should be uploaded under the name Assignment06_solution.pdf in the main branch of the assigned git repository.

Assignment 9 (Graded) (Total Points: 30)

1. Image-Based Visual Servoing with Point Features (6 points)

Question: Consider an image-based visual servoing controller using three point features. The current normalized image coordinates are $p_1 = (-0.2, -0.1)$, $p_2 = (0.3, -0.1)$, and $p_3 = (0.1, 0.4)$. The desired normalized image coordinates are $p_1^* = (-0.1, -0.05)$, $p_2^* = (0.2, -0.05)$, and $p_3^* = (0.0, 0.3)$. The feature vector is $s = [x_1, y_1, x_2, y_2, x_3, y_3]^\top$. Assume that only planar camera motion is allowed, with camera velocity $\xi = [v_x, v_y, \omega_z]^\top$. For each point feature, use the simplified interaction matrix $L_i = \begin{bmatrix} -1 & 0 & y_i \\ 0 & -1 & -x_i \end{bmatrix}$. The stacked interaction matrix is $L = [L_1^\top, L_2^\top, L_3^\top]^\top$. The IBVS control law is $\xi = -\lambda L^+ e$, where $e = s - s^*$, $\lambda = 0.5$, and $L^+ = (L^\top L)^{-1} L^\top$.

1. Compute the feature error vector $e = s - s^*$.

   (1 point)

2. Construct the stacked interaction matrix L.

   (2 points)

3. Compute $L^\top L$ and $L^\top e$.

   (2 points)

4. Compute the camera velocity command ξ.

   (1 point)

2. Interactive Scene Exploration with Two-Step Lookahead (4 points)

Question: A robot explores a cluttered shelf to improve its object map. It uses a two-step lookahead to compare two candidate action sequences. The first sequence is $\pi_1 = (\text{view}, \text{view})$. The second sequence is $\pi_2 = (\text{push}, \text{view})$. The robot evaluates each sequence using the score $J(\pi) = \Delta H_1 + \gamma \Delta H_2 - \alpha(R_1 + \gamma R_2)$, where ΔH_1 and ΔH_2 are the predicted entropy reductions after the first and second action, R_1 and R_2 are the corresponding risk costs, $\gamma = 0.8$ is the lookahead discount factor, and $\alpha = 0.5$ weights manipulation risk.

For the view-view sequence, the predicted values are $\Delta H_1 = 0.9$, $R_1 = 0.0$, $\Delta H_2 = 0.6$, and $R_2 = 0.0$. For the push-view sequence, the predicted values are $\Delta H_1 = 0.4$, $R_1 = 1.0$,

$\Delta H_2 = 2.0$, and $R_2 = 0.0$. The low first-step entropy reduction for pushing reflects that the map is not fully improved until the robot observes the scene again after the push.

1. Compute the score $J(\pi_1)$ for the view-view sequence.

   (1 point)

2. Compute the score $J(\pi_2)$ for the push-view sequence.

   (1 point)

3. Which sequence should the robot select according to the given objective?

   (1 point)

4. Explain why a two-step lookahead can select a different action than a one-step greedy strategy in interactive scene exploration.

   (1 point)

3. Rotation and Quaternions (5 points)

(a) Explain briefly why it is necessary to normalize a quaternion to represent a valid 3D rotation. What is the physical meaning of the conjugate of a unit quaternion?

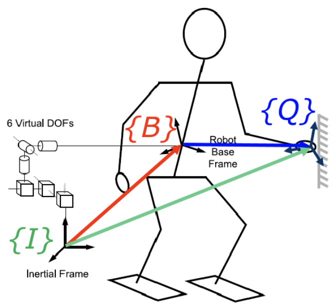
(2 points)

(b) Suppose we want to rotate a 3D vector $\mathbf{v} \in \mathbb{R}^3$ using a unit quaternion $\mathbf{q}$. Provide pseudocode showing how to apply the rotation.

(3 points)

4. Floating Base Kinematics (4 points)

Consider a floating-base robot with inertial frame $\{I\}$, base frame $\{B\}$, and end-effector frame $\{Q\}$,



Write the forward-kinematics expression for the position of the end-effector frame $\{Q\}$ with respect to the inertial frame $\{I\}$. Explain each term in the equation.

5. Humanoid Base State Estimation (11 points)

We wish to implement a linear Kalman filter for estimating the floating-base position and linear velocity of a humanoid robot using IMU readings and foot-kinematics measurements. The state vector is defined as

$$\mathbf{x} = \begin{bmatrix} \mathbf{p}_B \\ \mathbf{v}_B \\ \mathbf{p}_L \\ \mathbf{p}_R \end{bmatrix} \in \mathbb{R}^{12},$$

where the base position $\mathbf{p}_B \in \mathbb{R}^3$, the base linear velocity $\mathbf{v}_B \in \mathbb{R}^3$, and left and right foot positions $\mathbf{p}_L \in \mathbb{R}^3$, $\mathbf{p}_R \in \mathbb{R}^3$ are expressed in the world frame.

(a) In practice, the full floating-base configuration and velocity are not directly measured. Which sensor modalities are typically available on a legged robot? Briefly explain which quantities can be measured directly, and which base-state quantities must be estimated using state estimation? . (2 points)

(b) What are the main advantages and drawbacks of using an Extended Kalman Filter (EKF) for base-state estimation in legged robots? (3 points)

(c) Under which assumptions can we design a linear Kalman filter to estimate the robot's base position and linear velocity? (3 points)

(d) As a measurement, the forward kinematics of the contacting foot relative to the base is provided. Additionally, by assuming walking on the flat ground we introduce a pseudo measurement that constrains the foot's height (z_R) to zero. At a given time, only the right foot is in contact with the ground, so the measurement model is

$$\mathbf{y} = \begin{bmatrix} \mathbf{p}_B - \mathbf{p}_R \\ z_R \end{bmatrix} = H\,\mathbf{x},$$

where $\mathbf{p}_R = [x_R,\, y_R,\, z_R]^\top$.

Write the measurement matrix H explicitly and specify its dimensions. (3 points)