Motion Planning for Manipulation

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Humanoid Motion Planning

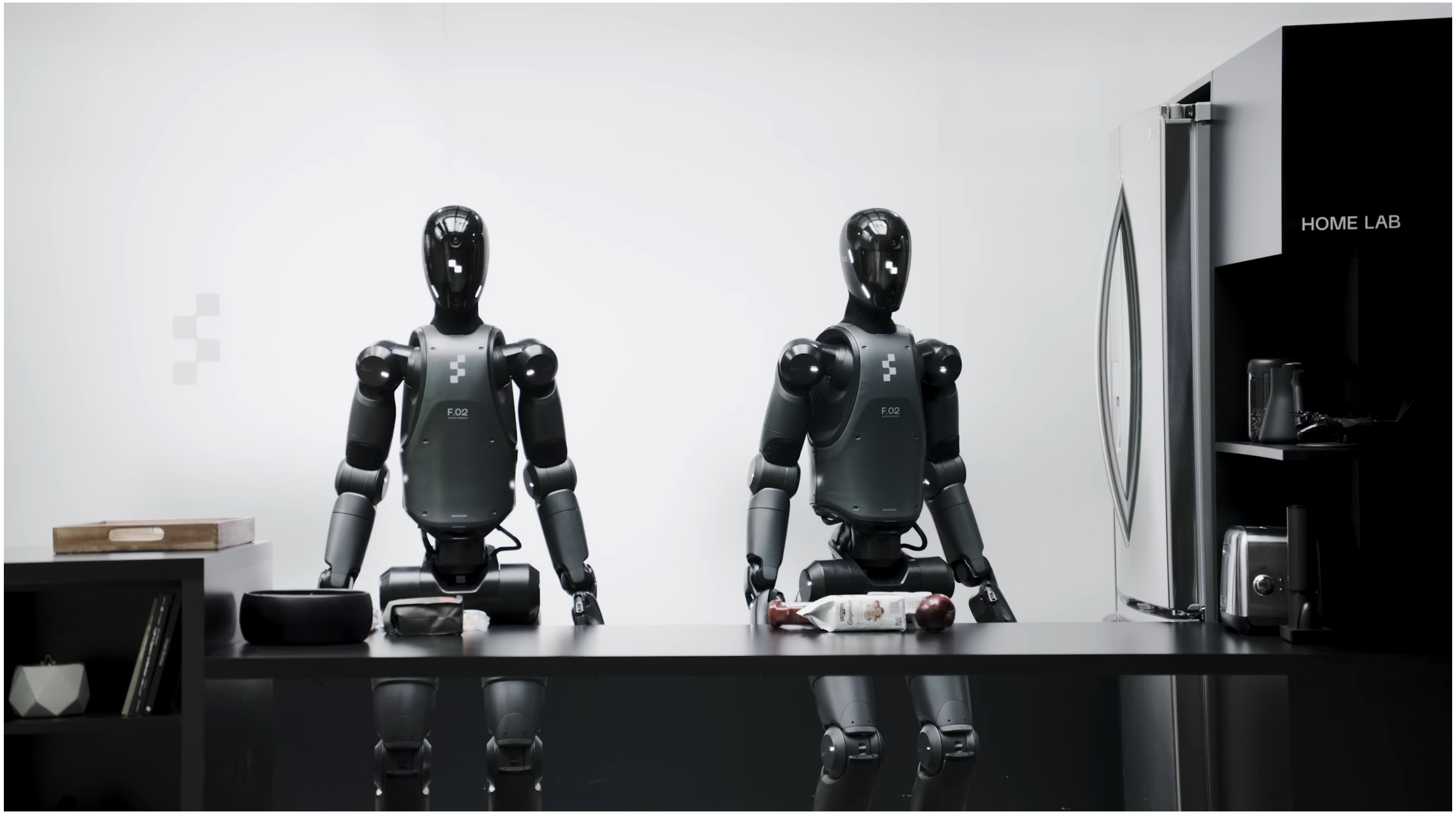
Different approaches based on the task

- Motion planning and trajectory generation for **manipulation** (upper limbs)
- Navigation and gait planning for **locomotion** (lower limbs)
- Whole-body controller ensures balancing during both tasks

Goal of This Chapter

- Introduction to basic concepts: path, trajectory, configuration space, task space
- Understanding of important components: configuration space obstacles, collision detection, sampling-based planning
- Next lecture (Tue June 3!): trajectory generation

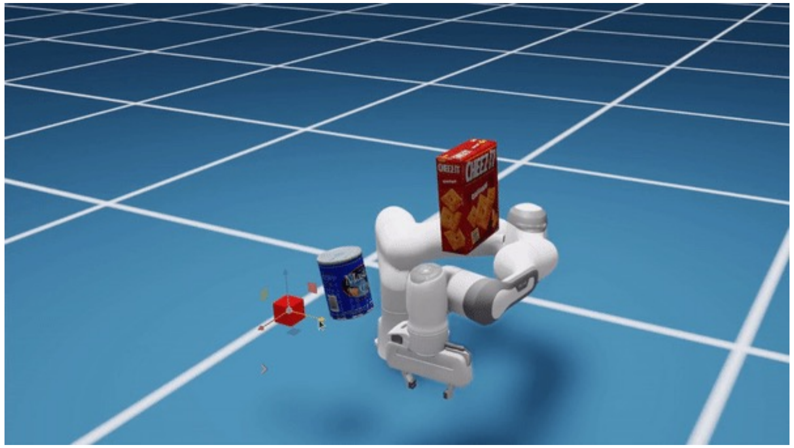
Motivation



Figure, “Introducing Helix”, 02/2025, www.youtube.com/watch?v=Z3yQHYNXPws

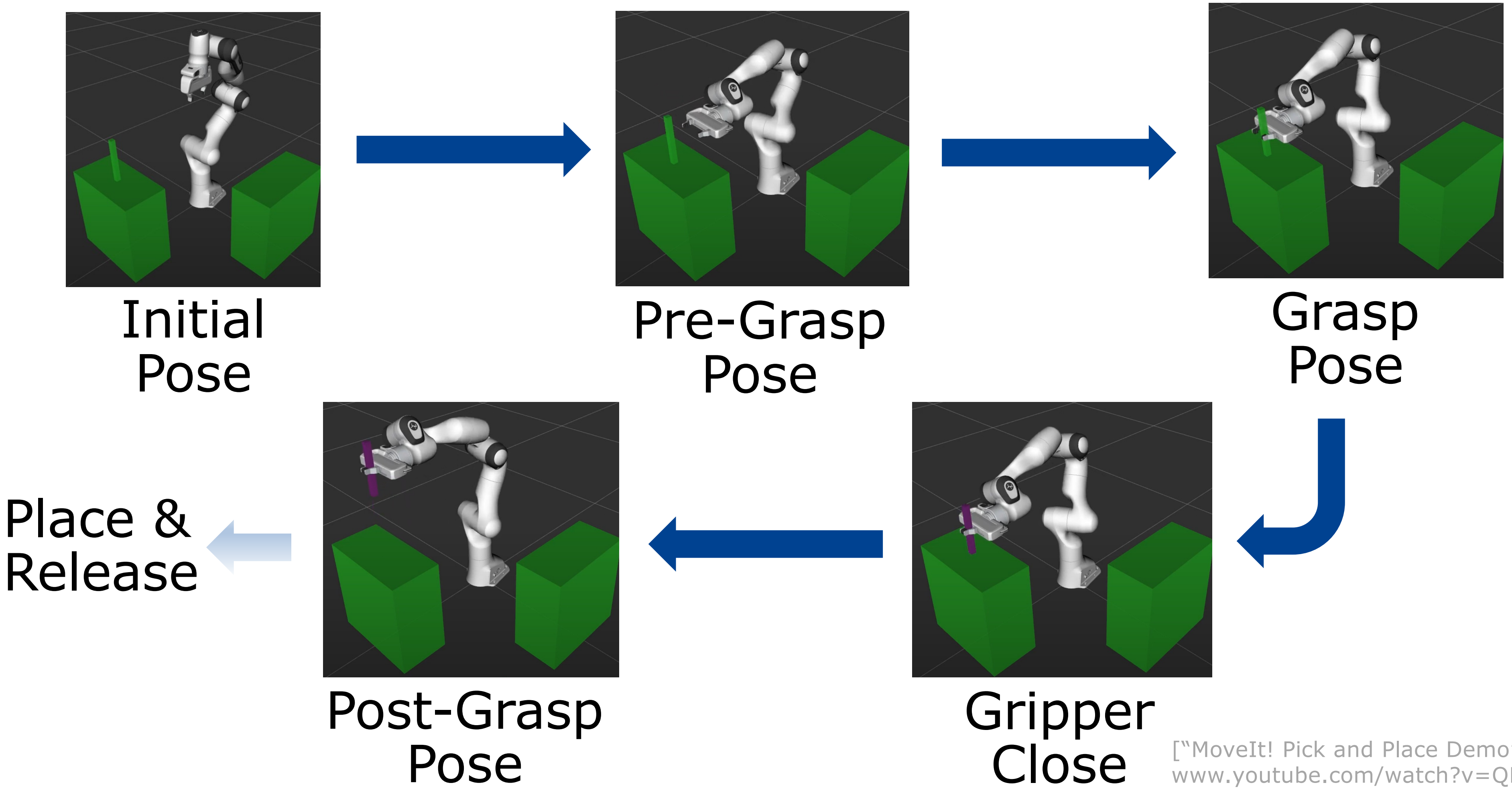
Motivation

- How to reach a target object such that the arm can manipulate it?
- How to reach such that the arm motion is **collision-free** in a cluttered environment?
- How to reach such that the arm motion is smooth **smooth**?
- How to reach such that the path obeys **temporal constraints**?



[Sundaralingam et al., "Curobo: Parallelized Collision-Free Robot Motion Generation", ICRA, 2023]

Motion Phases for Grasping Tasks

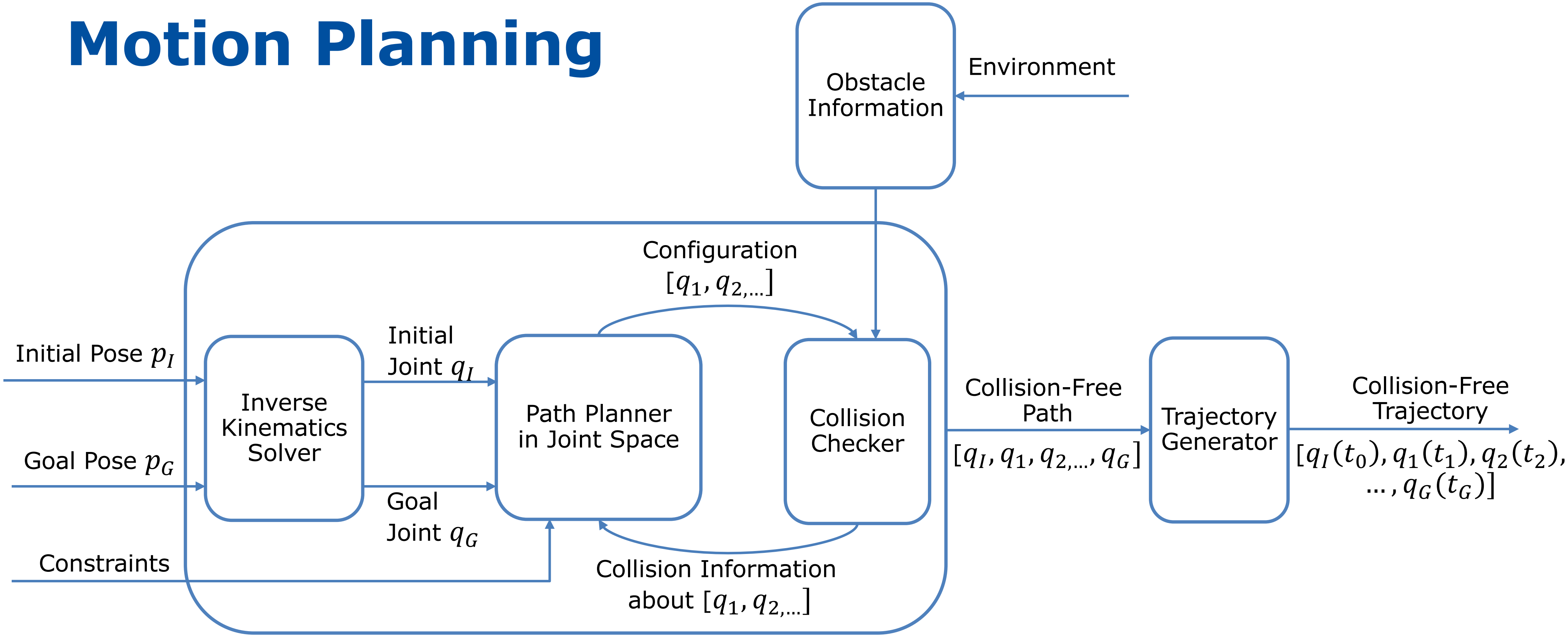


["MoveIt! Pick and Place Demo", 07/2018, www.youtube.com/watch?v=QBJPxx_63Bs]

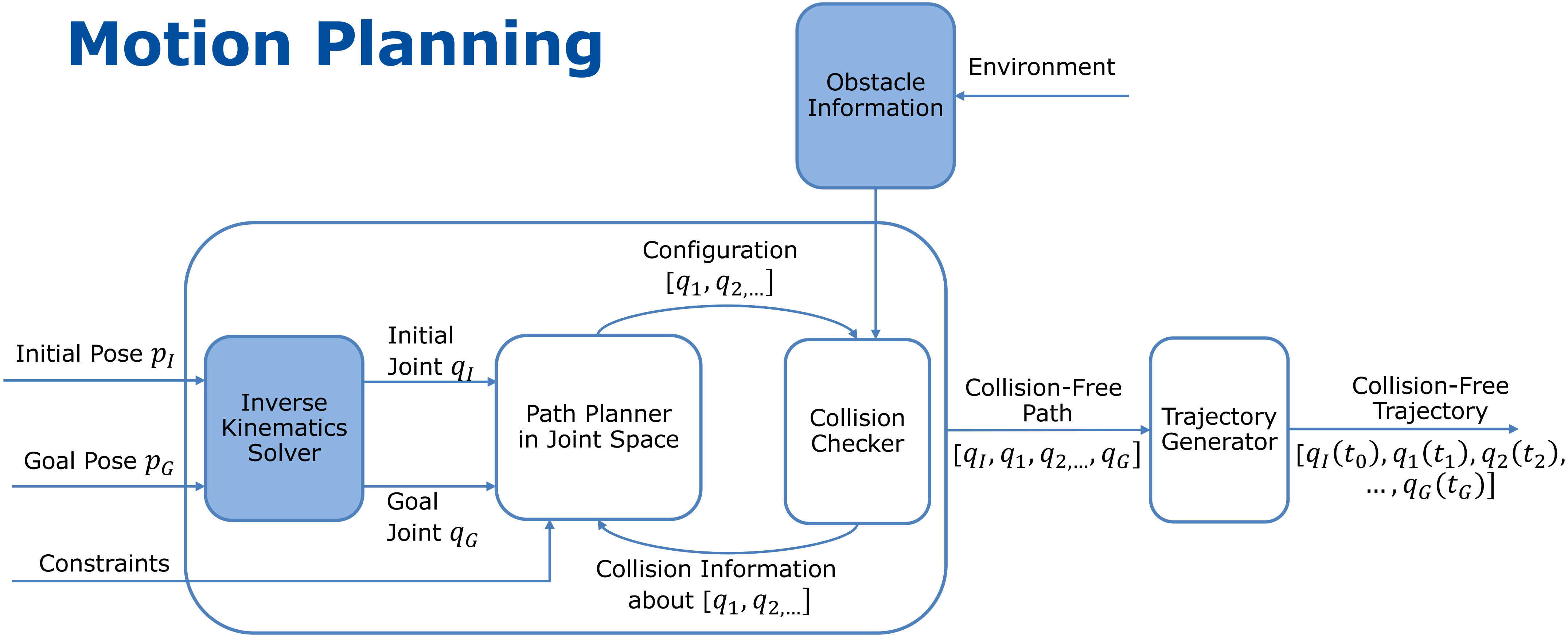
Steps in Manipulation Motion Planning

- Define start and goal end-effector poses
- Define intermediate poses if needed
- Add constraints if necessary
- Generate a collision-free arm motion path
- Parameterize a trajectory from the path

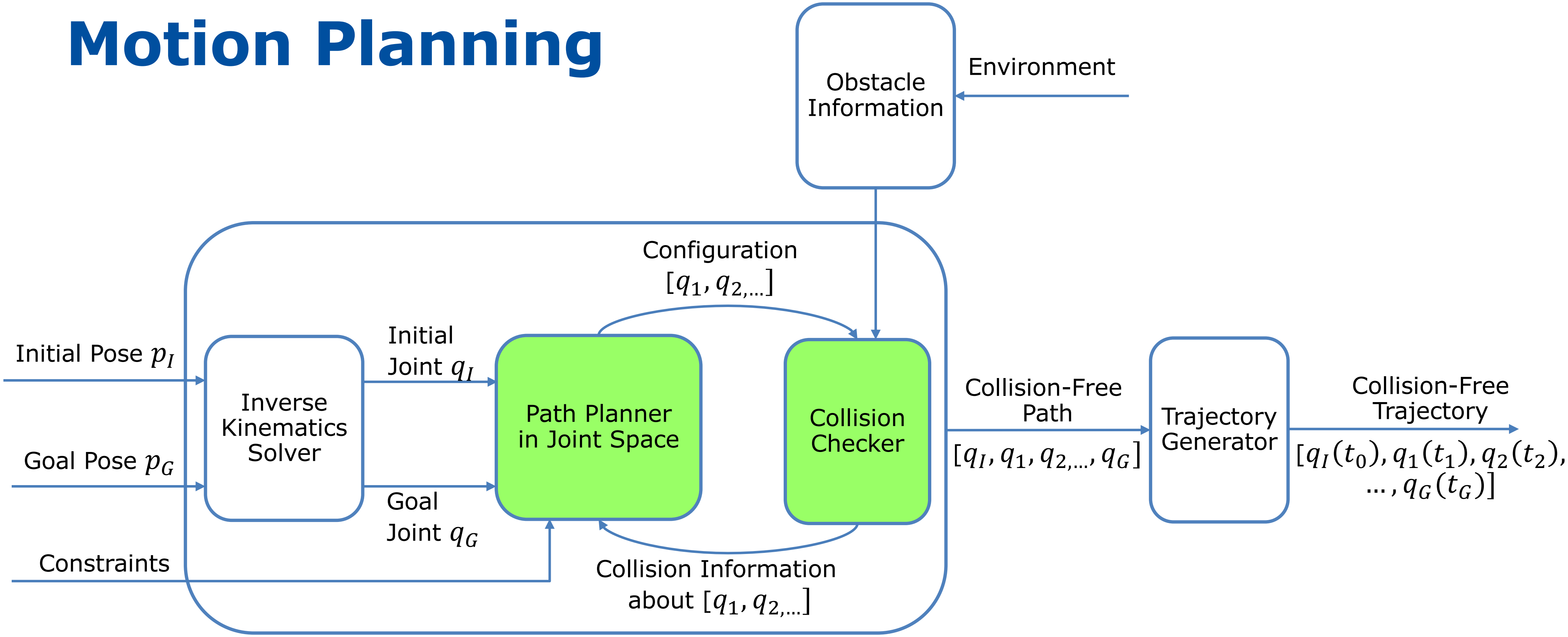
Motion Planning



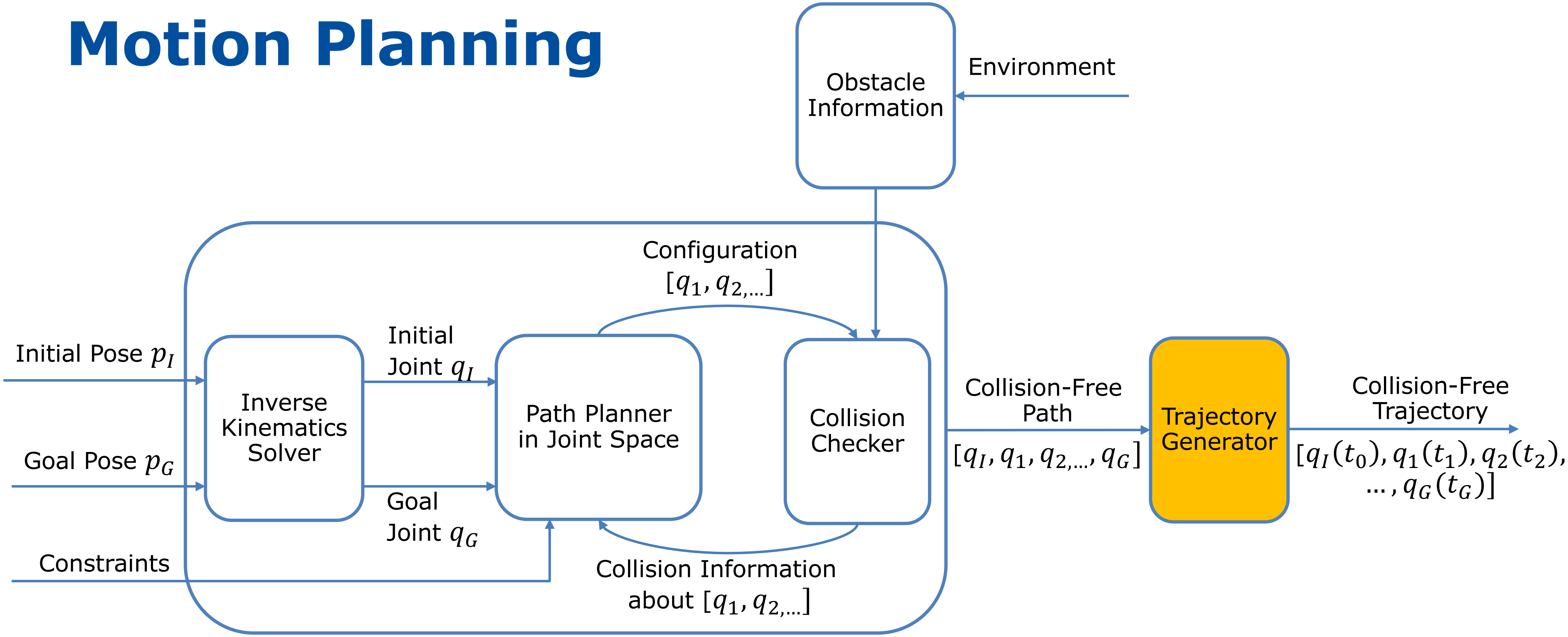
Motion Planning



Motion Planning



Motion Planning

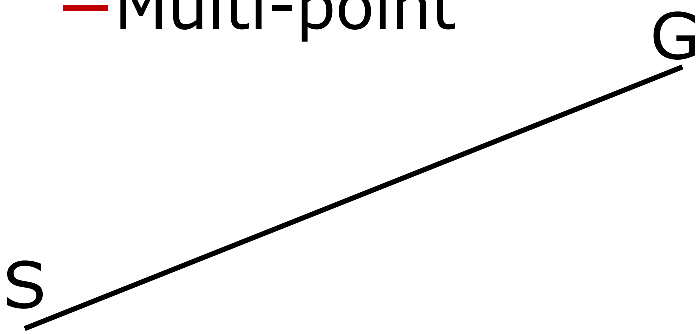


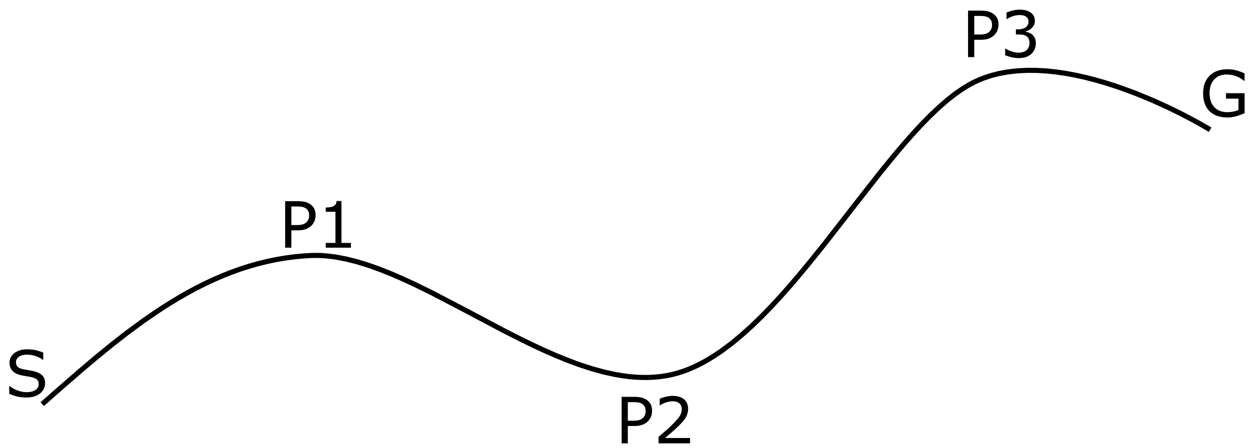
Concepts Needed for Motion Planning

- What is a path?
- What is a trajectory?
- What different kinds of robot spaces exist?
- How to plan a path?
- Ho to perform collision checking?
- How to generate a trajectory?

Path

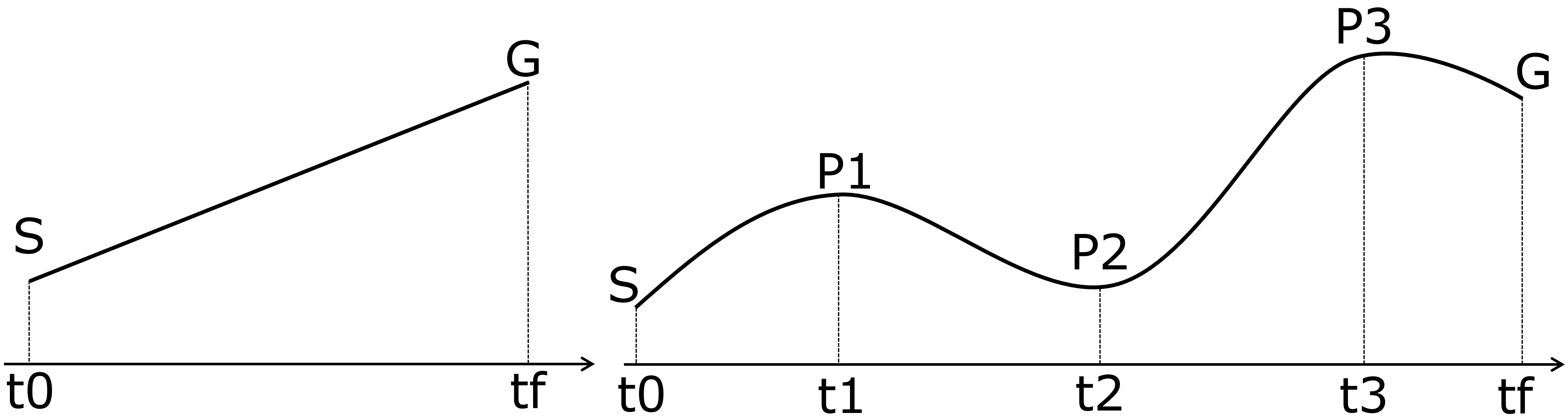
- Defines geometric sequence of positions
- Lacks timing and dynamic information
- Can be
  - Point to point
  - Multi-point





Trajectory

- Adds time parameterization to path
  - Initial and final times
  - Time optimality
- Specifies velocity, acceleration, jerk or torque along path



Robot Spaces

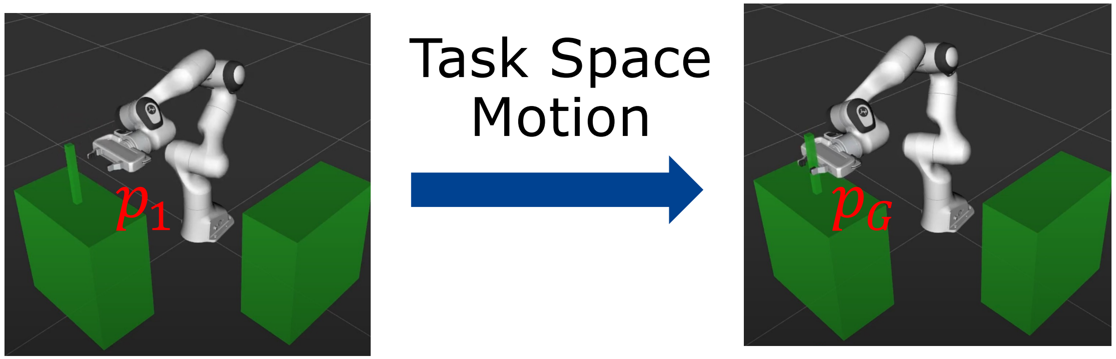
- Robots operate in multi-dimensional spaces
  - **Configuration space** (joint space): Space formed by the combination of robot joint angles
  - **Task space** (Cartesian space): Space described by end-effector position ($\mathbf{R}^3$) and orientation ($\mathbf{SO}(3)$), both ($\mathbf{SE}(3)$)
  - **Workspace**: Actual physical region that the end-effector can reach ($\mathbf{R}^2$ for mobile robot base, $\mathbf{R}^3$ for arms)
- Real-world tasks are specified in **task spaces**
- However, robots are controlled in **configuration space**
- Obstacle regions are typically given **workspace**

Task Space

- More intuitive than joint space for manipulation planning
- Controls end-effector pose (position, orientation)
- Enables direct control of robot's environmental interaction
- Crucial for grasping, tool use, and human-robot collaboration

Task Space Motion: Pre-Grasp to Grasp

- End-effector must linearly approach the object
- Interpolate in task space from p_1 to p_G keeping the gripper orientation fixed
- Compute inverse kinematics
  $[q_1 = IK(p_1), q_2 = IK(p_2), \ldots, q_G = IK(p_G)]$



Pre-Grasp Pose   Grasp Pose

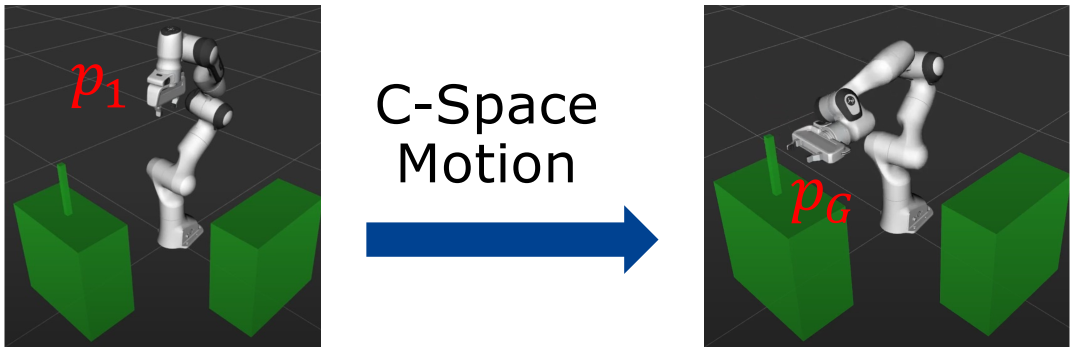
["MoveIt! Pick and Place Demo", 07/2018, www.youtube.com/watch?v=QBJPxx_63Bs]

Configuration Space (C-Space)

- Represents the space of the robot's joint angle configurations
- For a robot with n joints, its configuration space is an n-dimensional space
- High-dimensional, capturing all possible configurations
- Essential for collision checking and motion planning

Example: Initial to Pre-Grasp

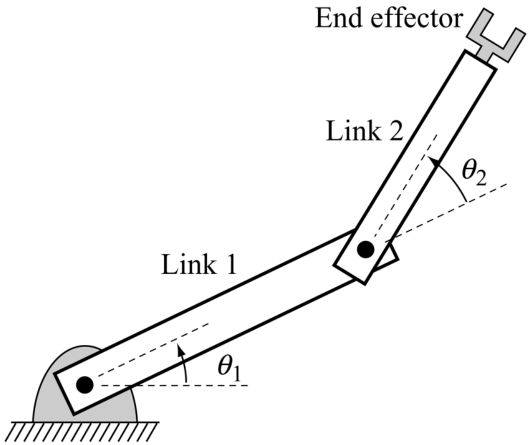
- Start and goal poses (p_1, p_G) defined in task space
- Compute inverse kinematics $q_1 = IK(p_1), q_G = IK(p_G)$
- Then, compute a path from q_1 to q_G in joint space
- In case of obstacles, generate a collision-free path $[q_1, q_2, \dots, q_G]$ in joint space

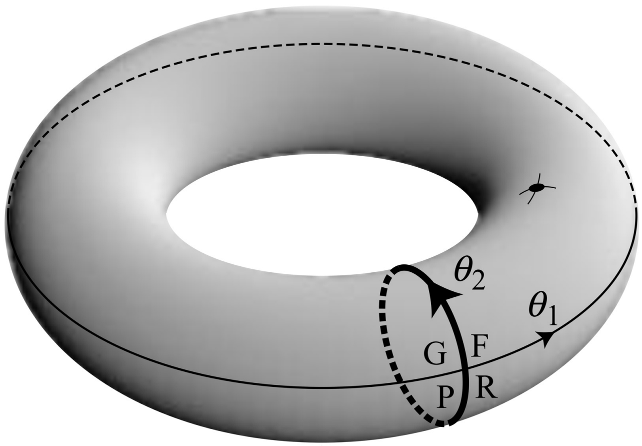


["MoveIt! Pick and Place Demo", 07/2018, www.youtube.com/watch?v=QBJPxx_63Bs]

C-Space of a Two-Joint Planar Arm

- Consider a 2-joint planar arm with no joint limits
- Each joint angle θ_i corresponds to a point on circle S^1
- C-Space is $S^1 \text{x}\, S^1 = T^2$corresponding to a 2D torus
- Configuration q in C-space consists of 2 angles $q = (\theta_1, \theta_2)$

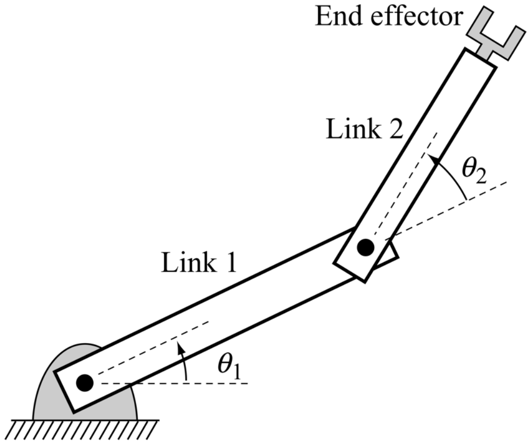


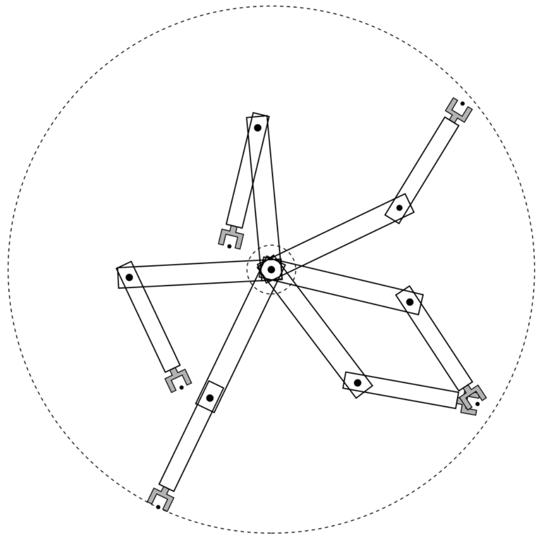


[Choset et al., *Principles of Robot Motion: Theory, Algorithms, and Implementations*, MIT press, 2005]

Workspace of a Two-Joint Planar Arm

- For the 2-joint planar arm, the workspace is a 2D torus, i.e., a subset of $\mathbb{R}^2$
- All points in the 2D torus are reachable with two different configurations: elbow-up or elbow-down





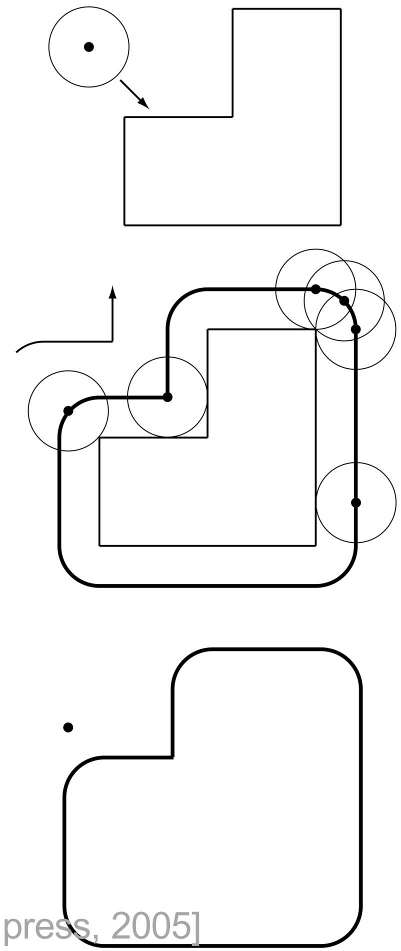
[Choset et al., *Principles of Robot Motion: Theory, Algorithms, and Implementations*, MIT press, 2005]

C-Space Obstacles and Free Space

- Typically, complete description of the robot's geometry and of its workspace W is provided
- Let $O \subset W$ represent the **workspace obstacle region**
- Let $A(q) \subset W$ denote set of points occupied by the robot when in configuration $q \in C$
- **C-space obstacle**: $C_{obs} = \{q \in C | A(q) \cap O \neq \emptyset\}$
- **Free C-space**: $C_{free} = C \setminus C_{obs}$

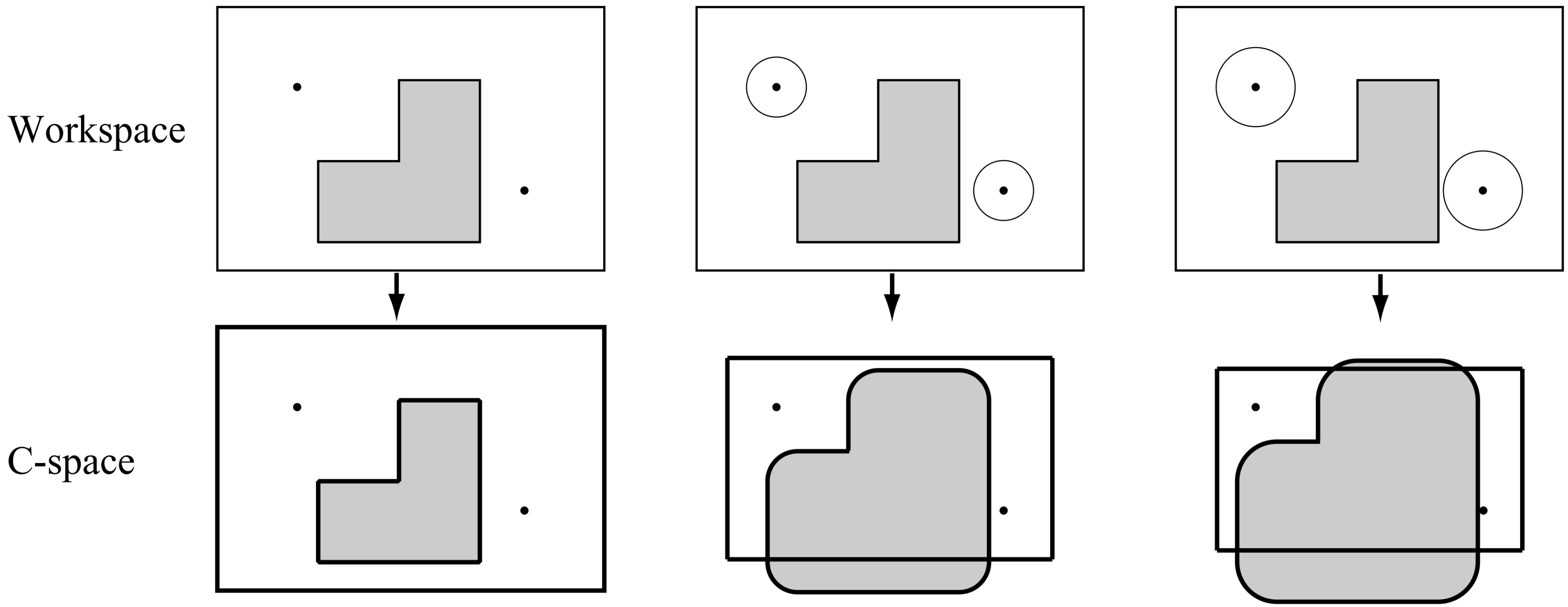
Workspace Obstacles to C-Space Obstacles

- Consider circular mobile robot with single polygonal obstacle as shown
- "Slide" the robot around the obstacle to find the constraints the obstacle places on the configuration of the robot
- Resulting obstacle in C-space
- Motion planning for circular robot in top figure is equivalent to motion planning for point in C-space



[Choset et al., *Principles of Robot Motion: Theory, Algorithms, and Implementations*, MIT press, 2005]

Workspace and C-Spaces for Different Mobile Robots



[Choset et al., *Principles of Robot Motion: Theory, Algorithms, and Implementations*, MIT press, 2005]

What about Transforming Workspace Obstacles to C-Space for n-Joint Arms?

- For circular mobile robots, converting workspace obstacles to C-space is relatively trivial due to
  - Symmetry of the robot
  - Workspace and C-space being low dimensional $\mathbb{R}^2$
- Robot arms have workspace in $\mathbb{R}^3$ and task space in SE(3)
- C-space is T^n with n number of joints
- Hence, conversion of workspace obstacles to C-space is computationally infeasible

Geometric Path Planning Problem

Given

- Robot's configuration space C
- Robot's workspace W
- Obstacle region $O \subset W$
- Initial configuration $\mathrm{q_I} \in C_{free}$
- Goal configuration $\mathrm{q_G} \in C_{free}$

Goal

For the query $(\mathrm{q_I}, \mathrm{q_G})$, compute a collision-free path $[q_I, q_1, q_{2,\ldots}, q_G]$ in the configuration space

Motion Planning Complexity

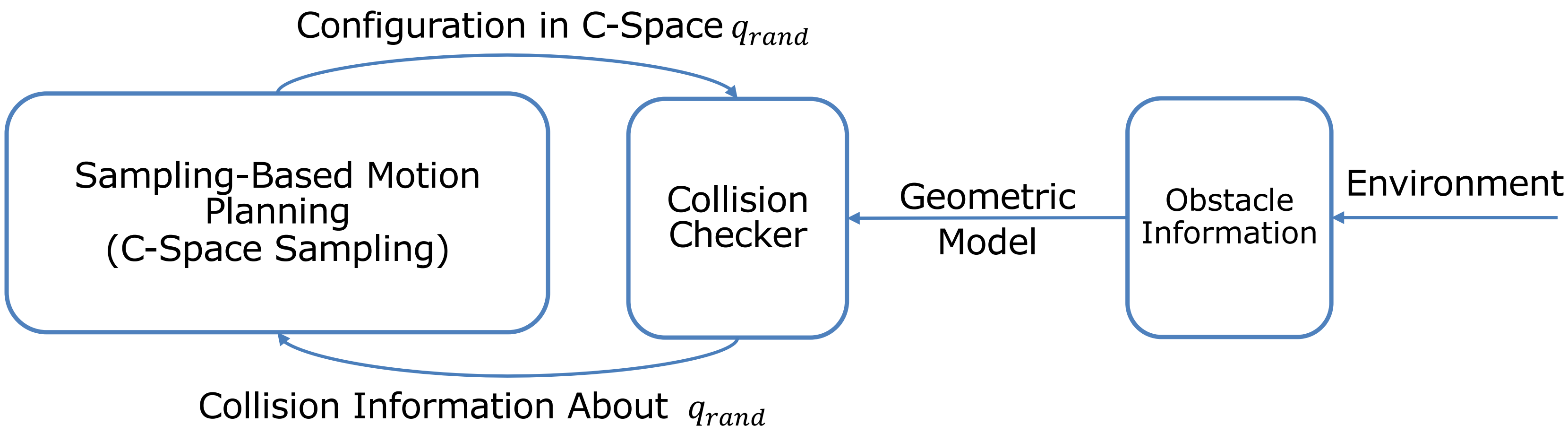
- Not easy to compute C_{obs} and C_{free}
- Exponential dependence on C-space dimensionality
- Two approaches: Combinatorial and Sampling
- Combinatorial algorithms
  - **Complete**, i.e., either find a solution or will correctly report that no solution exists
  - **Exact**, i.e., find paths through C-space w/o resorting to approximations
  - However, NP-hard

Motion Planning Complexity

- Sampling-based approach
  - **Weaker** guarantee: Will find a solution eventually if one exists, but no guarantee on failure report in finite time in case none exists
  - **Approximate**: Uses approximation of C-space for collision checking

Sampling-Based Motion Planning

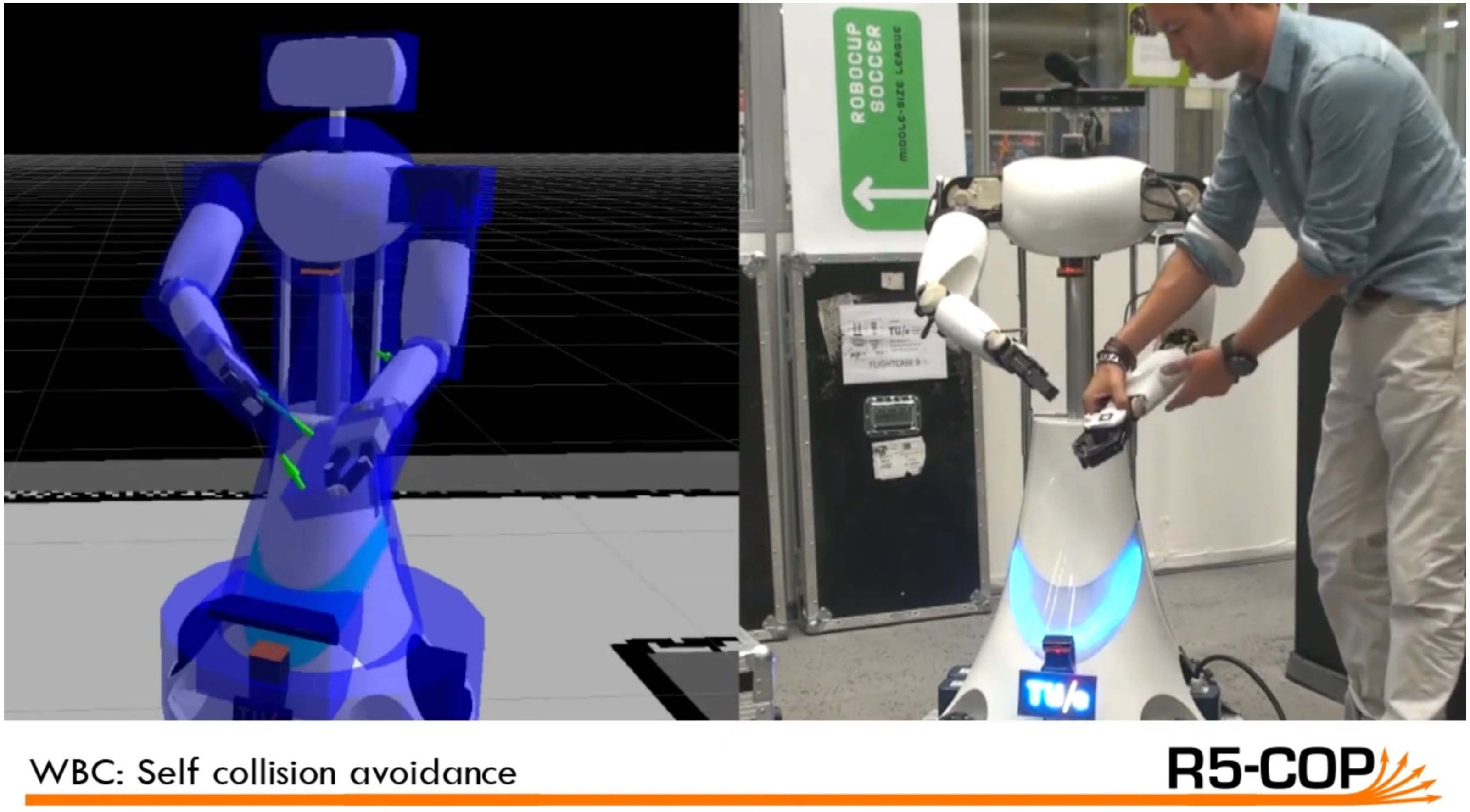
- Avoid explicit construction of the obstacle configuration space C_{obs}
- Instead, perform search that probes C-space with sampling
- Collision checking without exact geometric models



Geometric Models

- Representations for known objects, i.e., robot and known obstacles
  - Primitives (rectangle, cylinder, box, sphere)
  - Meshes
- Representations for unknown objects, i.e., sensed obstacles
  - Point clouds
  - Occupancy maps
- See Chapter 3 for more details on 3D world representations

Collision Avoidance



[Tech United Eindhoven, “Reactive Collision Avoidance With the AMIGO Robot”, 01/2016, www.youtube.com/watch?v=7GcLU9l65eM.]

Collision Detection

- For a particular configuration $q \in T^n$, check if $q \in C_{free}$ or $q \in C_{obs}$
- Collision detection can be a continuous or Boolean function
- Boolean function $\phi: C \rightarrow \{TRUE, FALSE\}$

$$q \in C_{obs} \rightarrow \phi(q) = TRUE, else\ FALSE$$

- Boolean functions typically used in sampling-based planners for accepting or rejecting a q sampled from T^n
- Distance function $d: C \rightarrow [0, \infty)$
- Distance function used for optimization-based planning where d is used to assign a cost for q

Two-Phase Collision Detection

- For *n*-joint robots like arms collision detection is a two-phase process
- **Broad Phase:**
  - Avoid expensive computation for links far away from each other
  - Place simple bounding boxes around each links
  - Perform simple overlap test to determine whether costly checking is needed

Two-Phase Collision Detection

- **Narrow Phase:**
  - Further process individual pairs of bodies that overlap in broad-phase check
  - Perform more expensive checking for collision

Sampling-Based Motion Planning

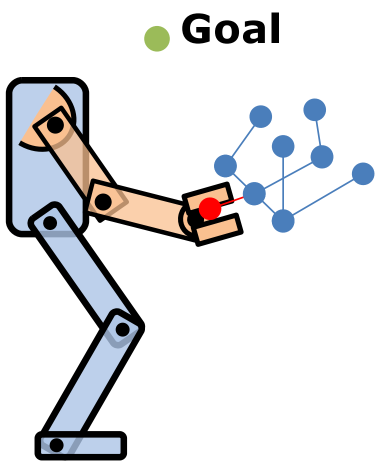
Different types of sampling-based planners

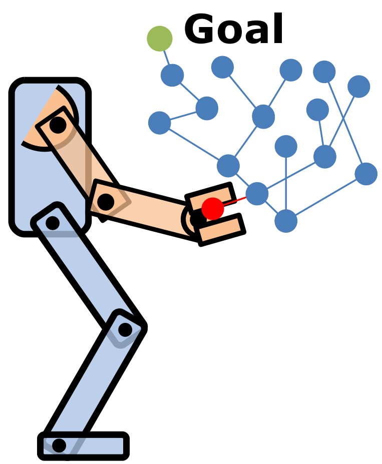
- Multi-query (e.g., probabilistic roadmap approach)
  - Constructs a “roadmap” once to map the C_{free}
  - Multiple queries in same environment using the roadmap

- Single-query (e.g., RRTs)
  - Build tree data structures on the fly for a given query
  - Explore part of C-space to solve specific query as fast as possible

Rapidly Exploring Random Trees (RRTs)

- Explore the configuration space by expanding incrementally from an initial configuration
- Explored space corresponds to a **tree rooted at the initial configuration**
- Basic principle: **Sample configuration** and compute **local connection to nearest neighbor**



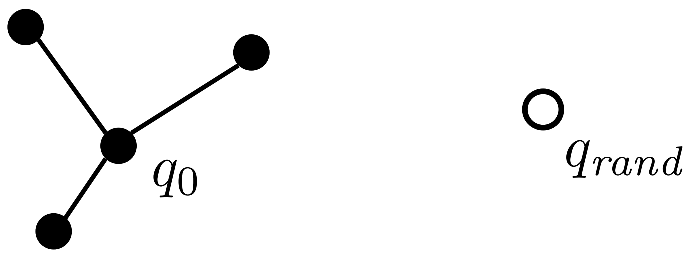


RRTs: General Algorithm

Given: Configuration space *C* and initial configuration q_0

G.init(q_0)
repeat
    $q_{rand} \rightarrow$ RANDOM_CONFIG($\mathcal{C}$)
    $q_{near} \leftarrow$ NEAREST(G, q_{rand})
    G.add_edge(q_{near}, q_{rand})
until *condition*

sample **random configuration**



tree constructed so far

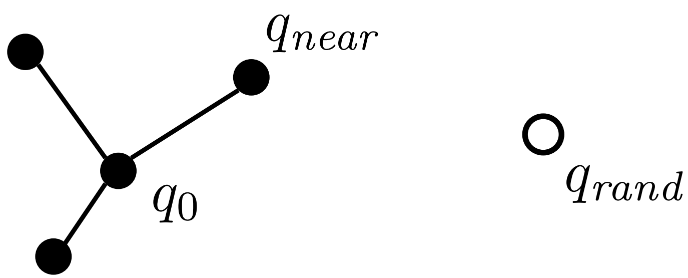
RRTs: General Algorithm

Given: Configuration space *C* and initial configuration q_0

$G.\text{init}(q_0)$
repeat
  $q_{rand} \rightarrow \text{RANDOM\_CONFIG}(\mathcal{C})$
  $q_{near} \leftarrow \text{NEAREST}(G, q_{rand})$
  $G.\text{add\_edge}(q_{near}, q_{rand})$
until *condition*

Find closest vertex in *G* using a **distance function**

$$\rho \; : \; \mathcal{C} \times \mathcal{C} \rightarrow [0, \infty)$$



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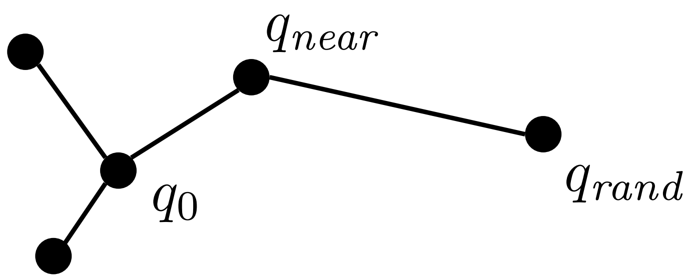
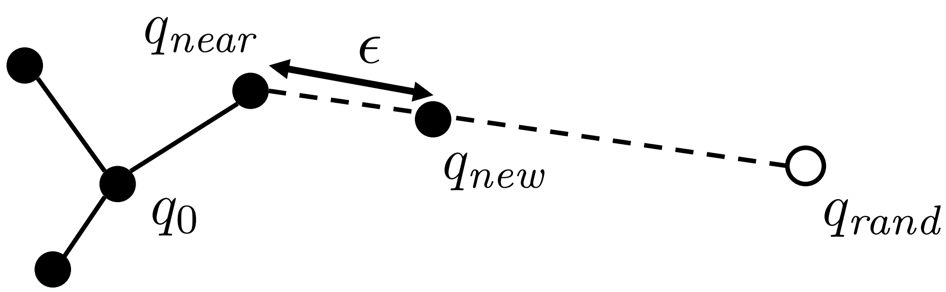
  $q_{near} \leftarrow$ NEAREST(G, q_{rand})
  G.add_edge(q_{near}, q_{rand})
until *condition*

Connect q_{near} with q_{rand} using a **local planner**



Extension of the Tree: Constraints

- Need to consider obstacles: Check local connection for collisions and add edge **only if path collision-free**
- Use **fixed incremental step size** so that the likelihood of a collision-free path is increased
- Terminate when q_{new} is close to the desired q_{goal}



Bias Towards the Goal

- During tree expansion, pick the goal instead of a random node with some probability (5-10%)
- Why not picking the goal at each iteration?
- Avoiding running into local minima (due to obstacles or other constraints) instead of exploring the space

Bidirectional RRTs

- High-dimensional, complex motion planning problems require more effective methods: **bidirectional search**
- Grow **two RRTs**, one from q_0 and one from q_G
- In every other step, try to extend each tree towards q_{new} of the other tree

RRT-Connect: Basic Concept

- Grow two trees: from start and end node (start and goal configurations of the robot)
- Pick a random configuration: q_{rand}
- Find the nearest node in one tree: q_{near}
- Extend the tree from the nearest node by taking a step towards the random node to get q_{new}
- Extend the other tree towards that q_{new} from nearest node in the tree
- Return the solution path when the distance between q_{new} and the nearest node in the second tree is close enough

Extend Function

Returns

- **Trapped**: Not possible to extend the tree due to collisions or constraints
- **Extended**: Performed a step from q_{near} towards q_{rand}, generated q_{new}
- **Reached**: Trees connected, path found

RRT-Connect

RRT_CONNECT ($q_{init,}$ q_{goal}) {
  $T_a.init(q_{init})$; $T_b.init(q_{goal})$;
  for k = 1 to K do
    q_{rand} = RANDOM_CONFIG();
    if not (EXTEND(T_a, q_{rand}) = Trapped) then
      if (EXTEND(T_b, q_{new}) = Reached) then
        Return PATH(T_a, T_b);
    SWAP(T_a, T_b);
  Return Failure;
}

K=max number of iterations

First tree has been extended, try to extend second tree

Success: trees connected

Max number of iterations reached

[Kuffner&Lavalle, ICRA 2000]

RRTs – Properties (1)

- Good balance between greedy search and exploration
- Effective for high-dimensional configuration spaces
- Produce non-optimal paths: solutions are typically jagged and may be overly long
- Post-processing such as smoothing is necessary
- Generated paths are not repeatable and unpredictable
- Rely on a distance metric (e.g., Euclidean)

RRTs – Properties (2)

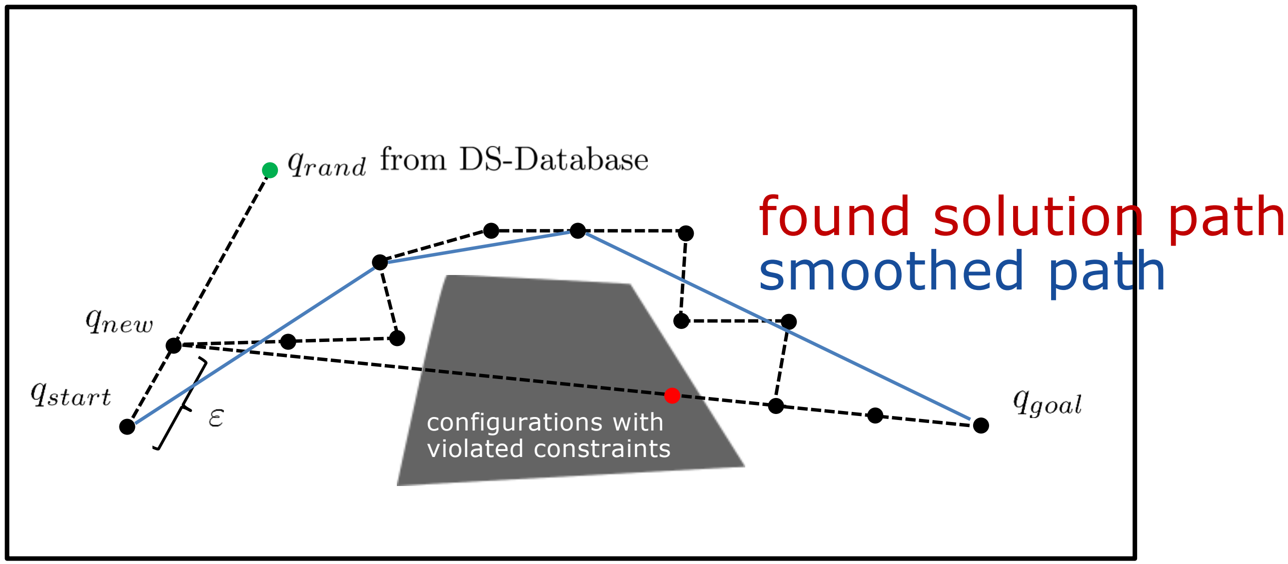
- Probability of finding a solution if one exists approaches 1 (probabilistic completeness)
- Unknown rate of convergence
- When there is no solution (path is blocked due to obstacles or other constraints), the planner may run forever
- To avoid endless runtime, the search is stopped after a certain number of iterations

Considering Constraints for Humanoid Motion Planning

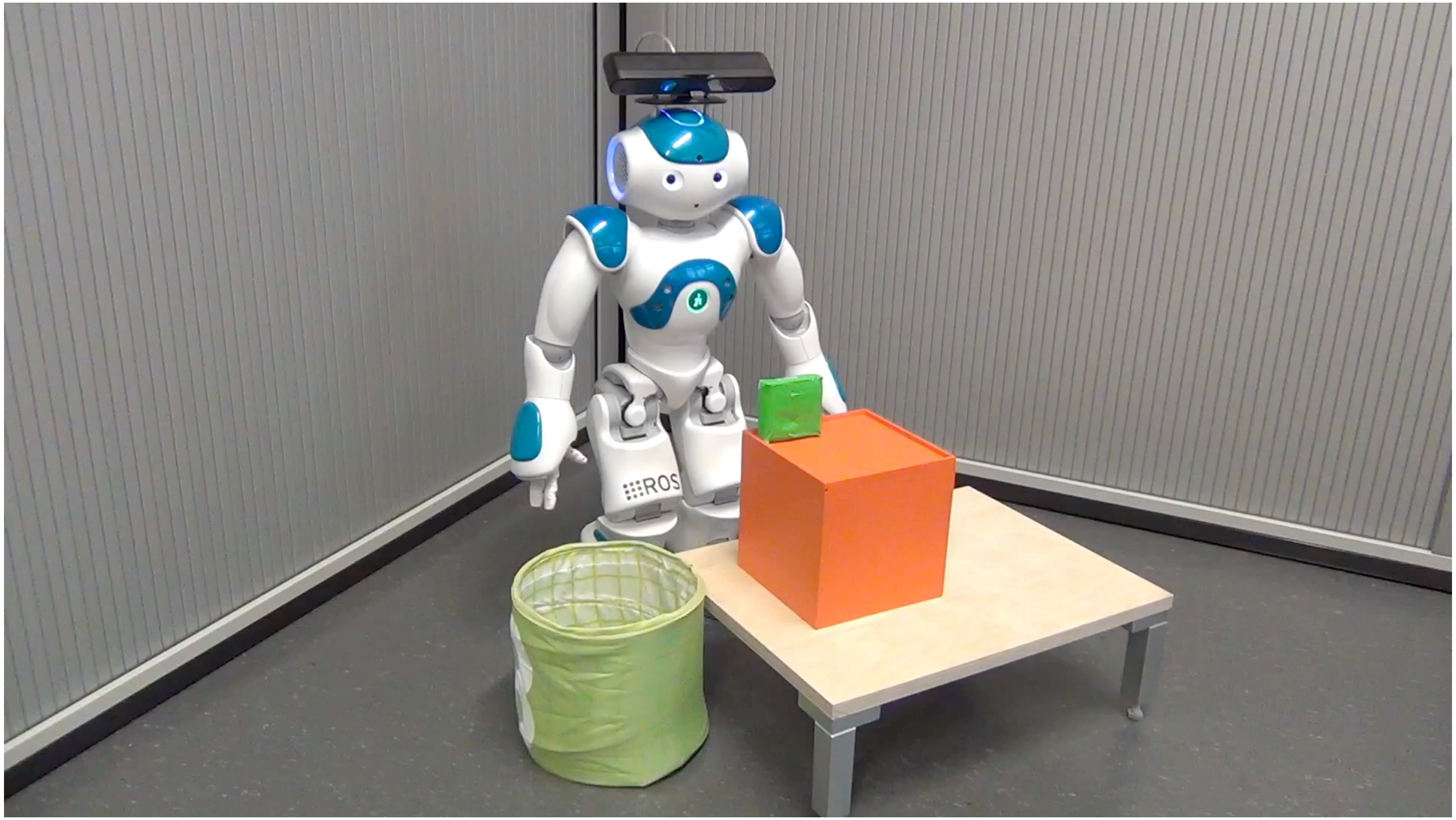
- When randomly sampling configurations, most of them will not be valid since they cause the robot to lose its balance
- Use a set of predetermined statically stable double support configurations from which to sample q_{rand}
- In the extend function: Check q_{new} for **joint limits, self-collision, collision with obstacles, and whether it is statically stable**

RRT-Connect: Considering Constraints

- Check for constraint violation in configuration space
- Smooth path after a solution is found



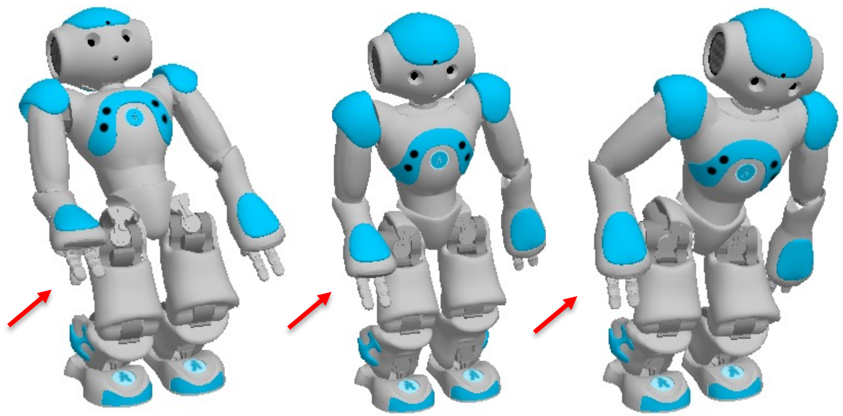
Path Execution: Pick and Place



Past Execution: Grabbing Into a Cabinet

Goal Configuration

- How to actually determine the robot's goal configuration for a given manipulation task?
- Use inverse reachability maps (see previous chapter)



all valid goal configurations
for the same desired end effector pose

Literature Motion Planning

- *Principles of Robot Motion: Theory, Algorithms, and Implementations,*
  Choset, Lynch, Hutchinson, Kantor, and Burgard, MIT press, 2005
- *Planning Algorithms,* LaValle, Cambridge University Press, 2006
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- *Curobo: Parallelized Collision-Free Robot Motion Generation,*
  Sundaralingam, Hari, Fishman, Garrett, Van Wyk, Blukis, Millane, Oleynikova, Handa, Ramos, and Ratliff, IEEE/RAS Int. Conf. on Robotics and Automation (ICRA), 2023
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- *RRT-Connect: An Efficient Approach to Single-Query Path Planning*
  Kuffner and LaValle , IEEE International Conference on Robotics & Automation (ICRA), 2000
- *Whole-Body Motion Planning for Manipulation of Articulated Objects*
  Burget, Hornung, and Bennewitz,
  IEEE International Conference on Robotics & Automation (ICRA), 2013

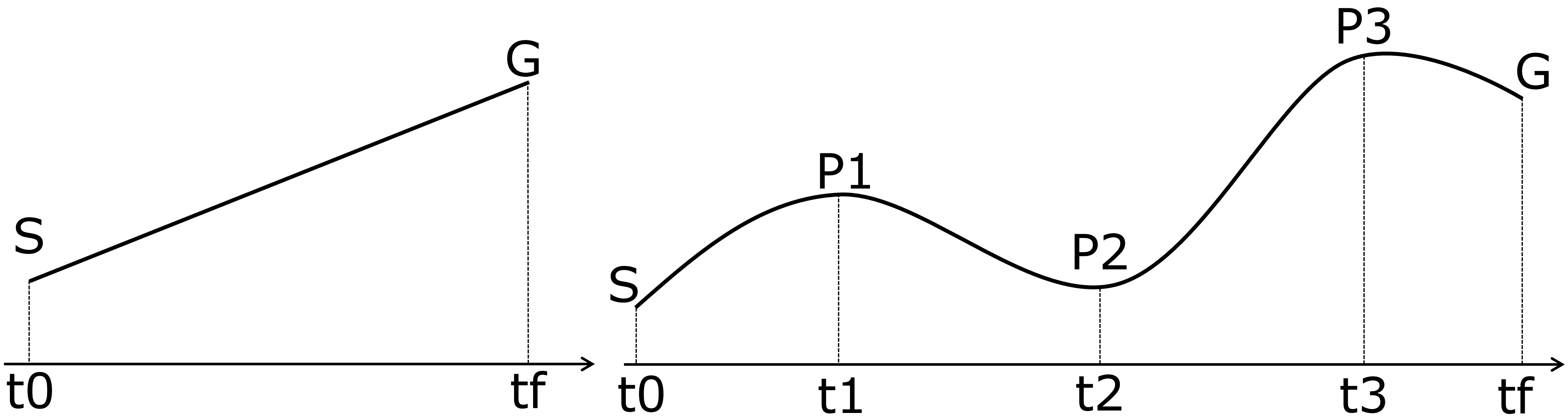
Trajectory Generation

Motivation

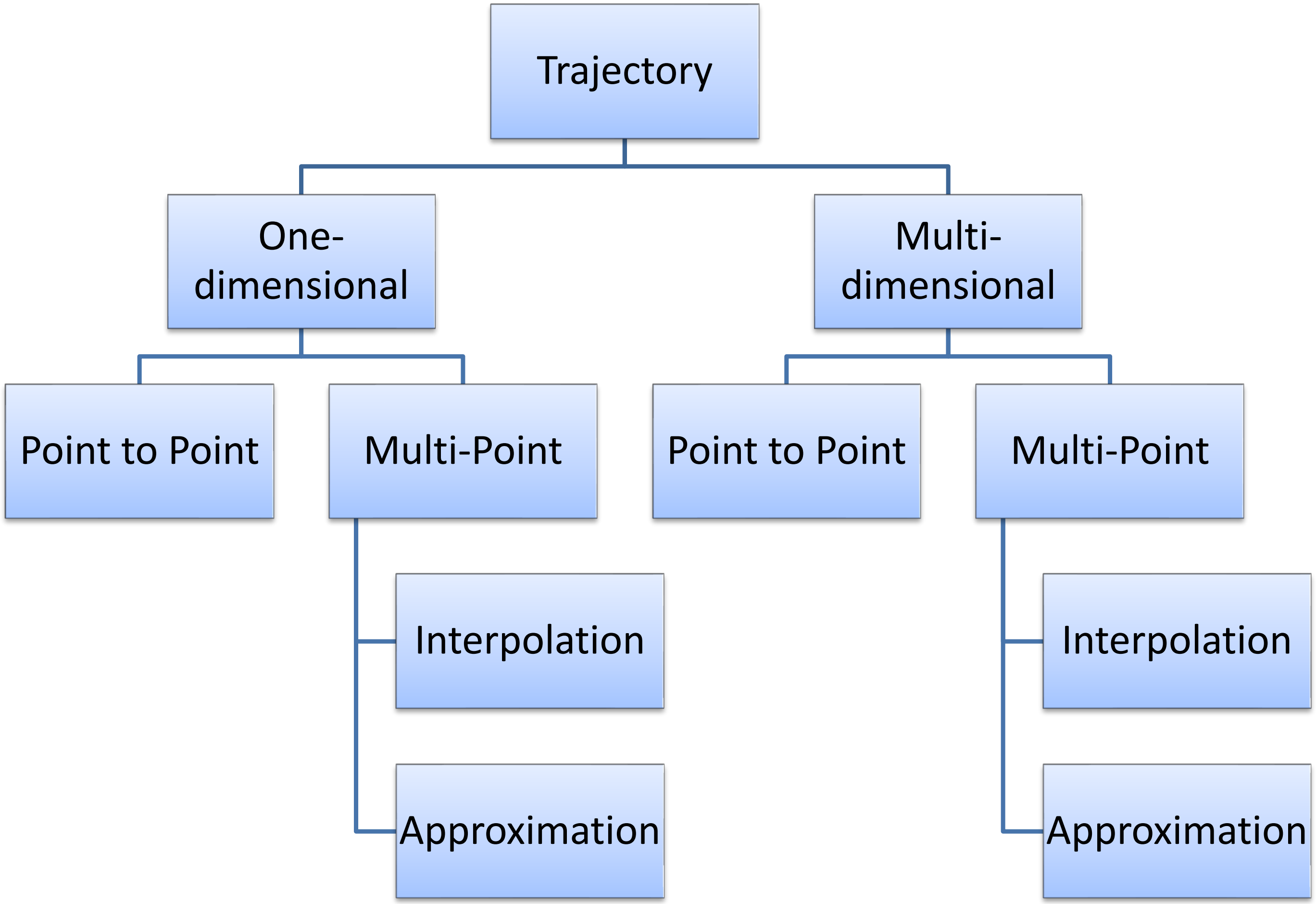
- Sampling-based planning typically produces only paths
- Assumes static obstacles and finds a collision-free path
- However, robots exist in the **spatial** and **temporal** world
- Paths need to be **parameterized with time**
- The optimal trajectory depends on the **constraints** on the velocities, torques, etc. of the robot joints

Trajectory

- Adds time parameterization to path
  - Initial and final times
  - Time optimality
- Specifies velocity, acceleration, jerk or torque along path



Main Trajectory Categories



General Objectives of Trajectory Parameterization in Joint Space

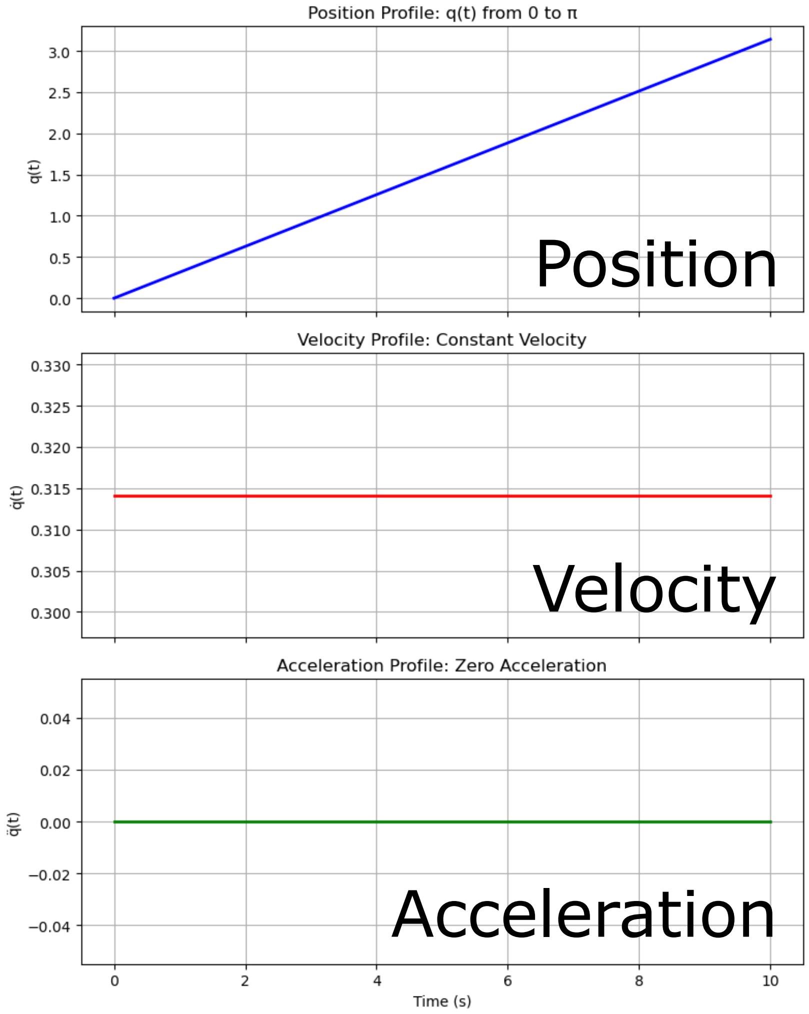
- Joint positions (q) and velocities ($\dot{q}$) should be differentiable
- Joint accelerations ($\ddot{q}$) should be at least continuous, or preferably differentiable for minimum jerk
- At the same time, the trajectory should be time-optimal

Point-to-Point Motion

- Define initial and final positions: q_0 at t_0 and q_1 at t_1 (position along computed path)
- No intermediate waypoints
- Specify start and end constraints
- Consider acceleration boundaries for smooth motion
- Basis for more complex motion profiles

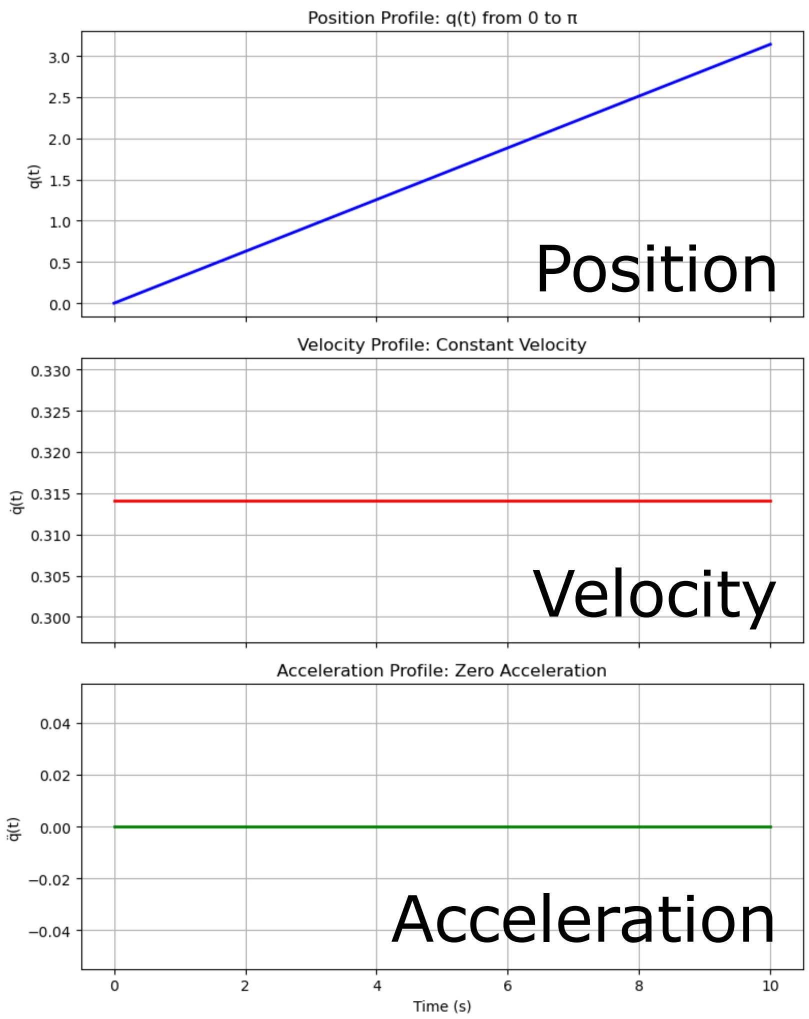
Constant Velocity Profile

- Most simple profile
- Linear
- $q(t) = a_0 + a_1(t - t_0)$
- We can specify either time constraints or maximum velocity



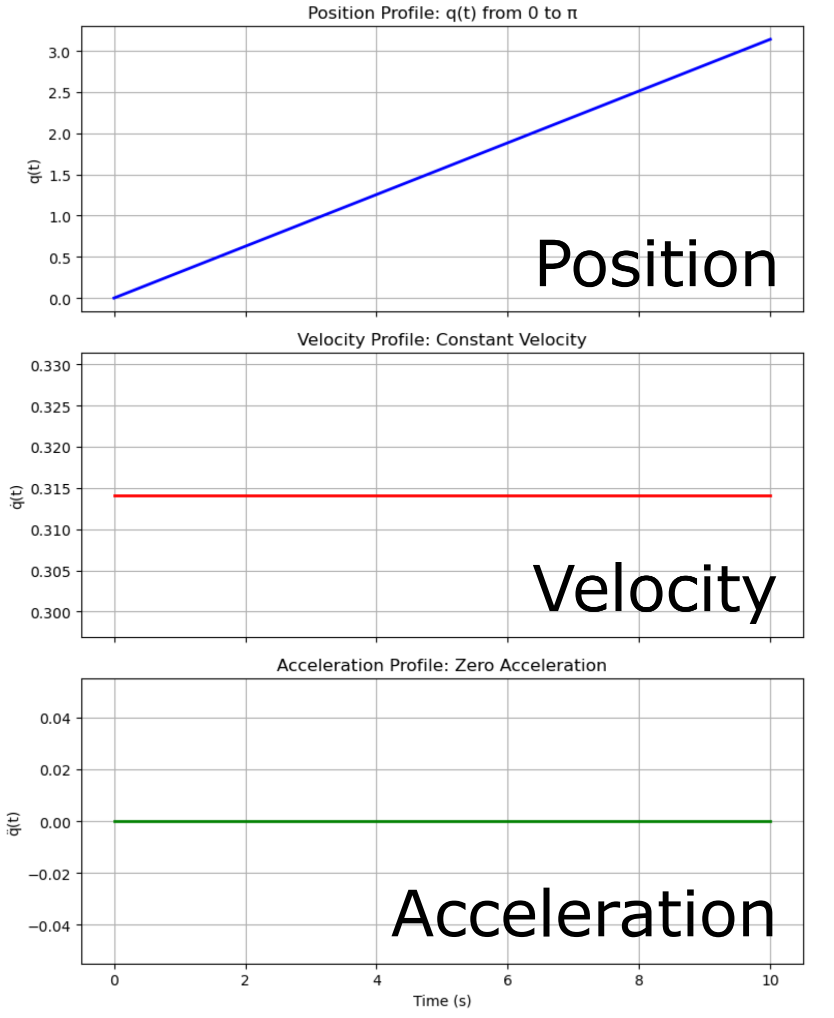
Constant Velocity Profile

- $q(t) = a_0 + a_1(t - t_0)$
- Consider time constraints
- Position q_0 at initial time t_0
- Position q_1 at final time t_1



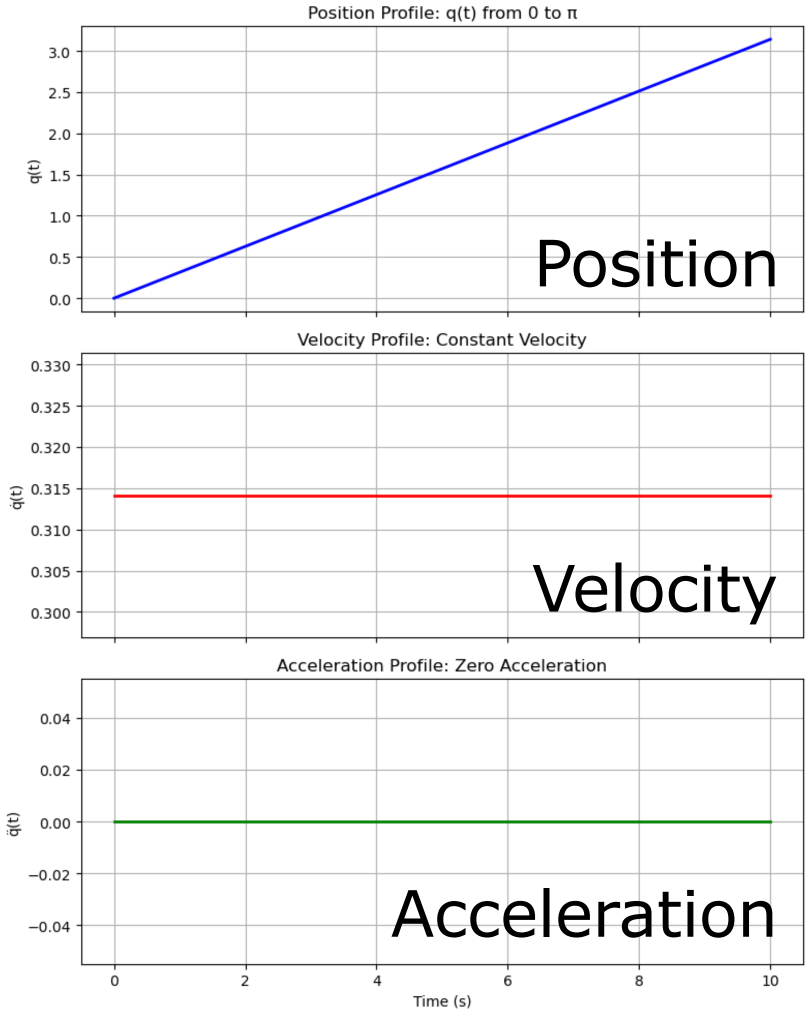
Constant Velocity Profile

- $q(t) = a_0 + a_1(t - t_0)$
- From position q_0 at initial time t_0
- $q(t_0) = q_0 = a_0$
- $a_0 = q_0$



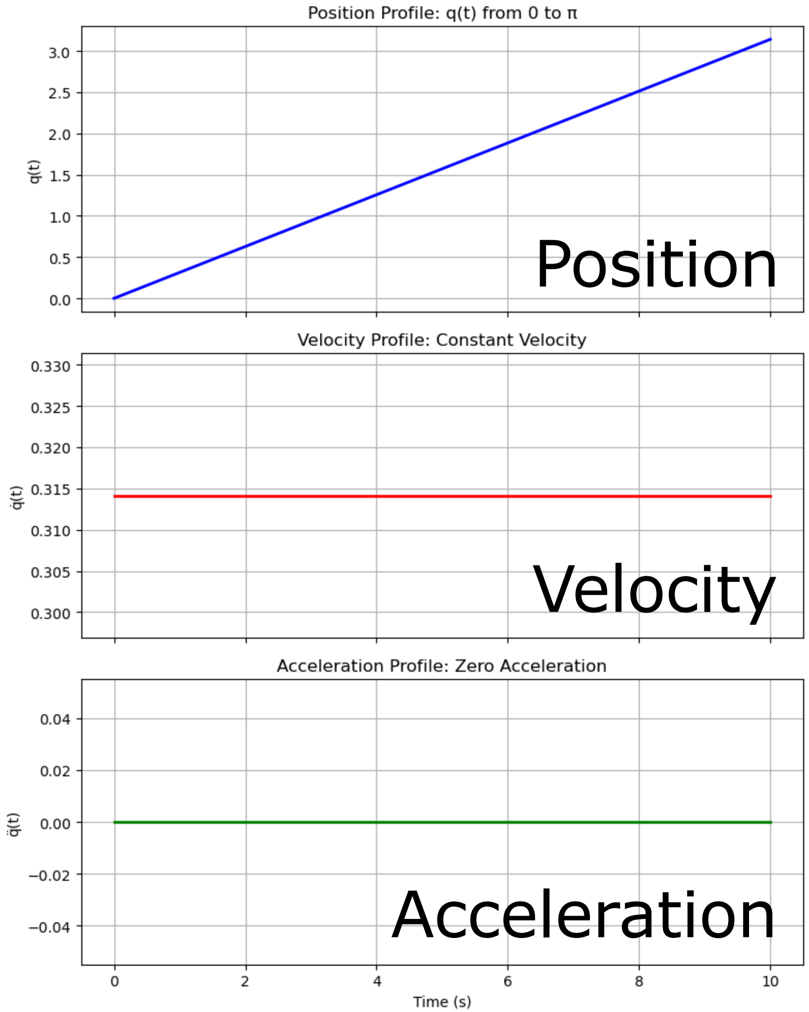
Constant Velocity Profile

- $q(t) = a_0 + a_1(t - t_0)$
- To position q_1 at final time t_1
- $q(t_1) = q_1 = a_0 + a_1(t_1 - t_0)$
- $a_1 = \frac{q_1 - q_0}{t_1 - t_0} = \frac{\Delta q}{\Delta t}$
- $\dot{q}(t) = \frac{\Delta q}{\Delta t} = a_1$ (constant velocity)



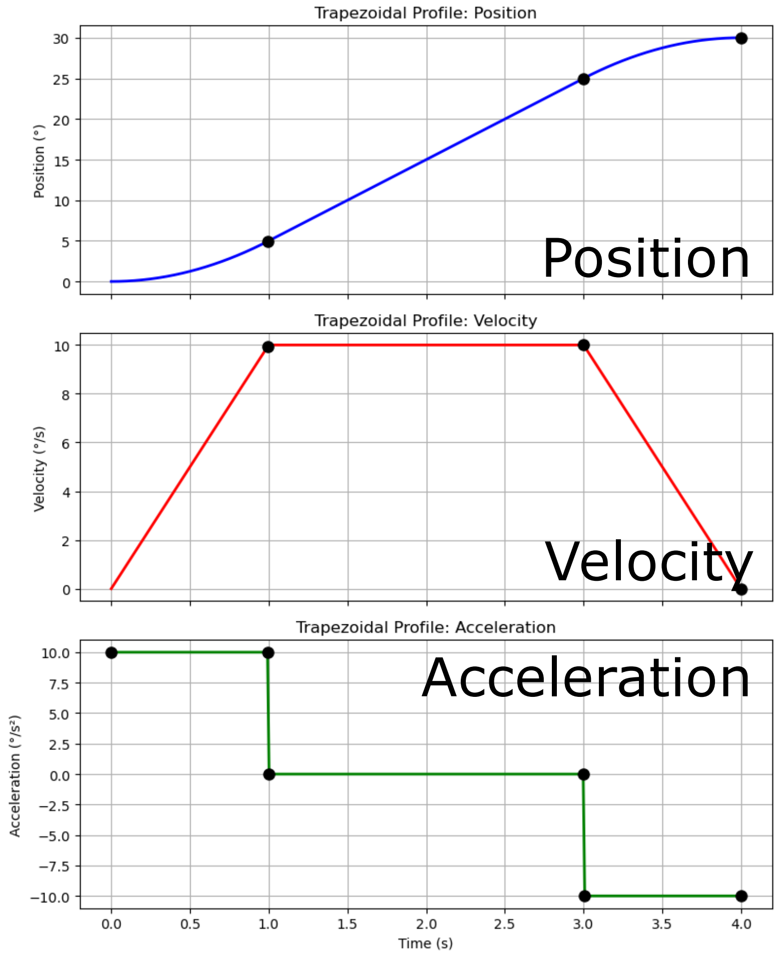
Constant Velocity Profile

- $q(t) = a_0 + a_1(t - t_0)$
- $\dot{q}(t) = \frac{\Delta q}{\Delta t} = a_1$ (constant velocity)
- Acceleration $\ddot{q}(t) = 0$ for $t_0 < t < t_1$
- However, $\ddot{q}(t)$ is undefined for $t = t_0, t = t_1$
- Leads to jerks at the start and end
- Introduces heavy loads on the actuators



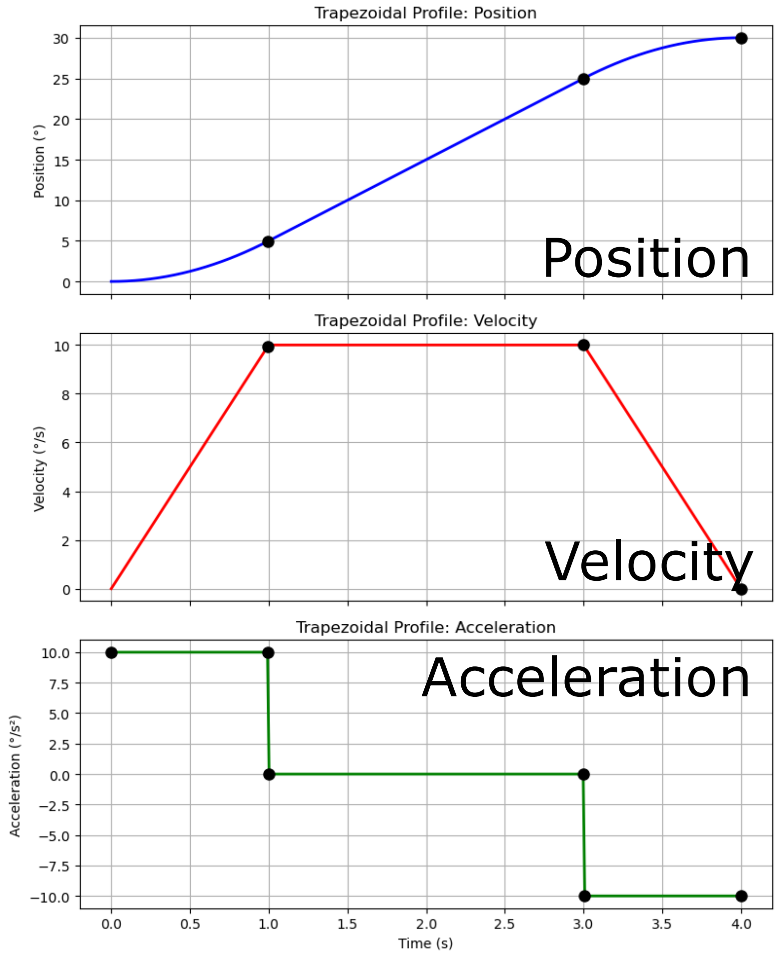
Trapezoidal Profile

- Fast, simple, and widely used
- Respects velocity and acceleration limits
- Motion divided into 3 phases
  - Acceleration phase
  - Constant velocity phase
  - Deceleration phase
- If motion duration too low, then no constant velocity phase



Trapezoidal Profile – Acceleration Phase

- Acceleration $t \in [0, T_a]$
- $q_a(t) = a_0 + a_1 t + a_2 t^2$
- $\dot{q}_a(t) = a_1 + 2a_2 t$
- $\ddot{q}_a(t) = 2a_2$
- a_0, a_1, a_2 defined by the constraints on initial position q_0 and velocity v_0, and the maximum velocity v_m
- $a_0 = q_0, a_1 = 0, a_2 = \frac{v_m}{2T_a}$



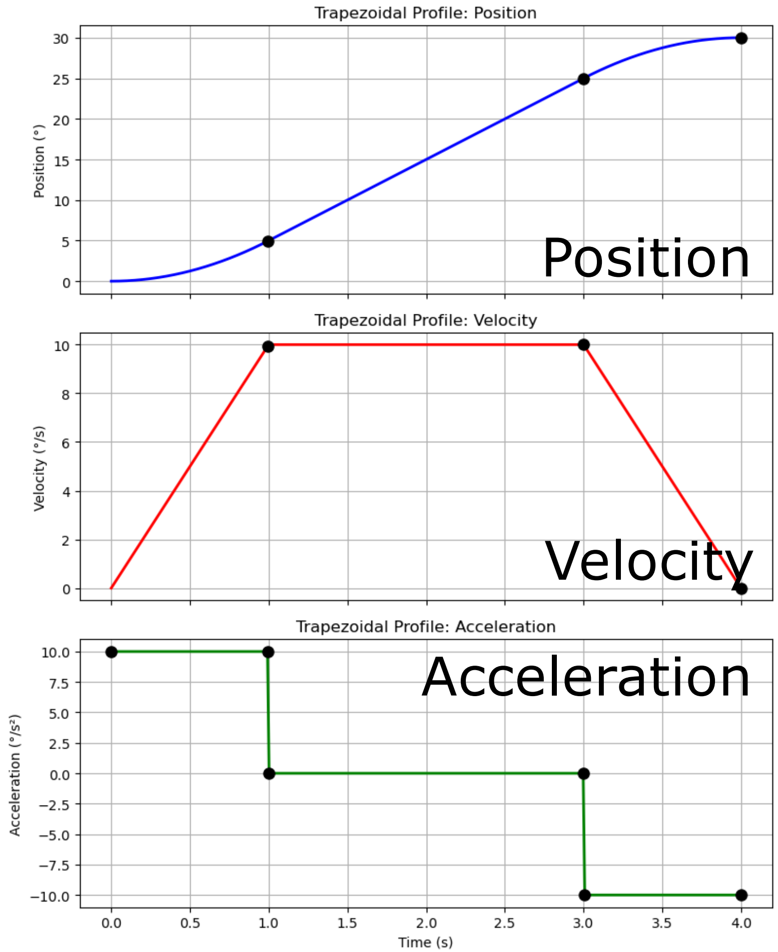
Trapezoidal Profile – Const. Velocity Phase

- Constant velocity $t \in [T_a, t_1 - T_a]$
- $q_c(t) = c_0 + c_1 t$
- $\dot{q}_c(t) = c_1$
- $\ddot{q}_c(t) = 0$
- For velocity to be continuous,

$$\dot{q}_c(T_a) = \dot{q}_a(T_a) \rightarrow c_1 = v_m$$

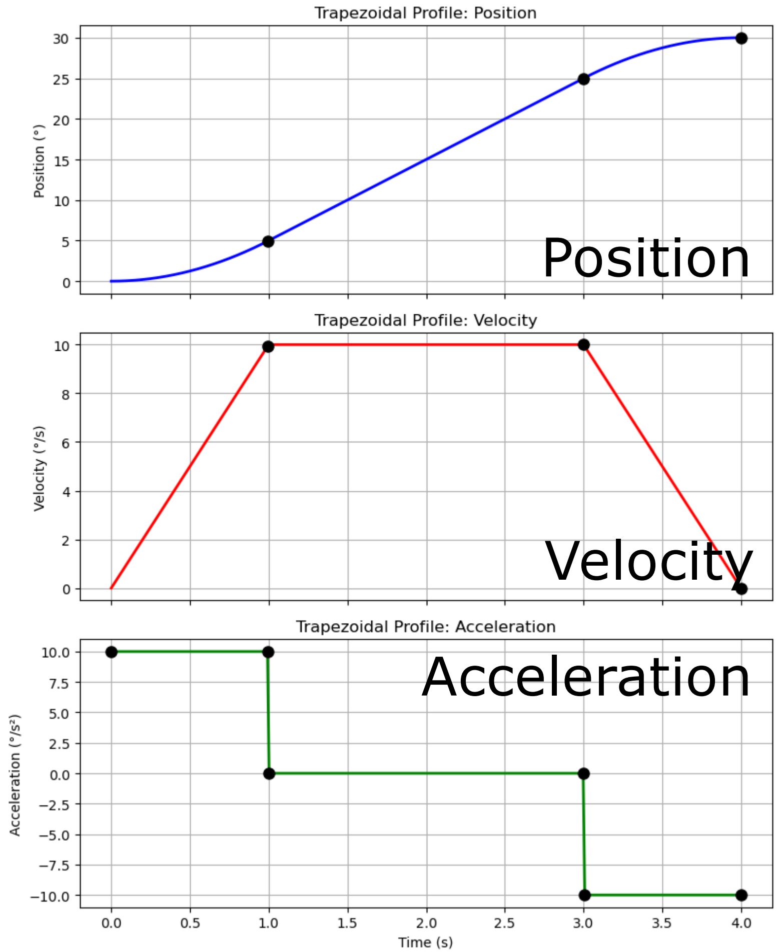
- For position to be continuous

$$q_c(T_a) = q_a(T_a) \rightarrow c_0 = q_0 - \frac{v_m T_a}{2}$$



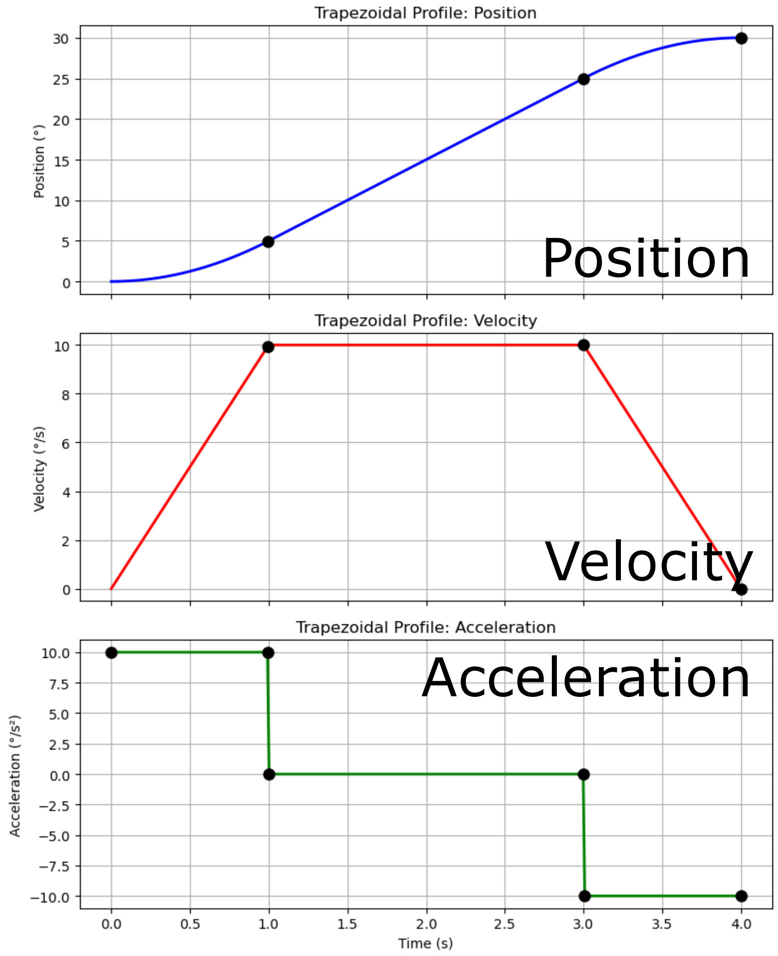
Trapezoidal Profile – Deceleration Phase

- Braking/deceleration $t \in [t_1 - T_a, t_1]$
- $q_b(t) = b_0 + b_1 t + b_2 t^2$
- $\dot{q}_b(t) = b_1 + 2b_2 t$
- $\ddot{q}_b(t) = 2b_2$
- With $\dot{q}_b(t_1) = 0, \dot{q}_b(t_1 - T_a) = \dot{q}_c(t_1 - T_a)$, $q_b(t_1 - T_a) = q_c(t_1 - T_a)$ and given q_1
- $b_0 = q_1 - \frac{v_m t_1^2}{2T_a}$, $\mathrm{b}_1 = \frac{v_m t_1}{T_a}$, $\mathrm{b}_2 = -\frac{v_m}{2T_a}$



Trapezoidal Profile

- Derived trajectory in position for all 3 phases
- Trapezoidal profile is commonly used in many robotic systems
- T_a determined in different ways
  - Maximum velocity constraint
  - Maximum acceleration constraint



Multi-Dimensional Trajectories

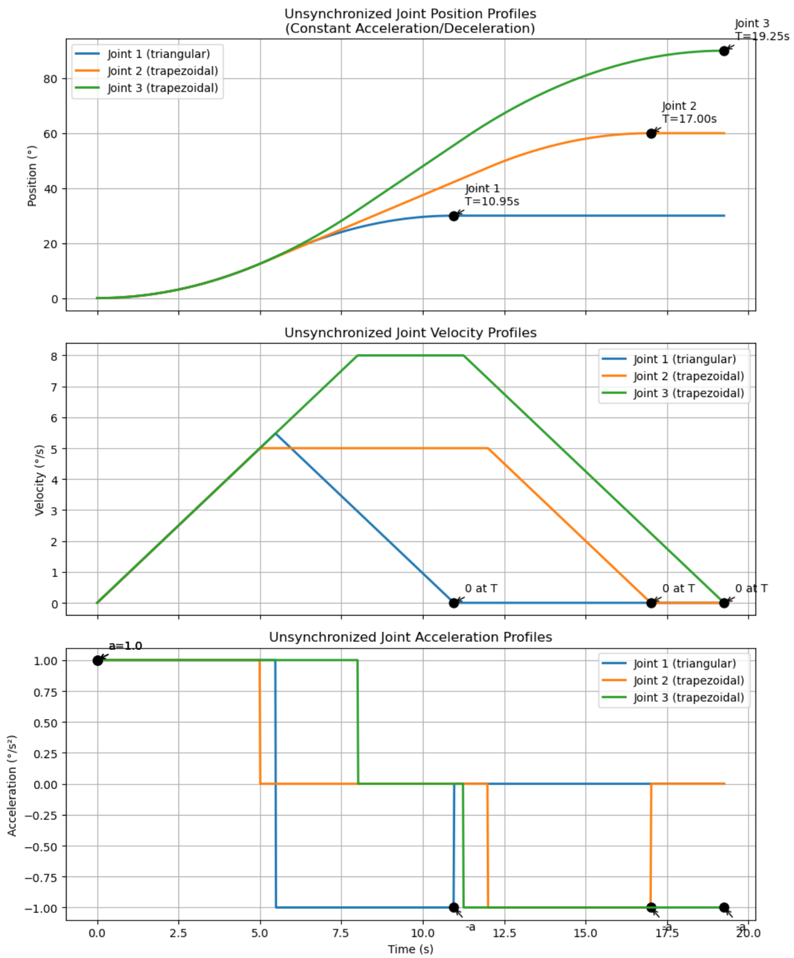
- Robotic manipulators have multiple degrees of freedom
- Typical robot trajectories span high-dimensional coordinated motions
- All joint motions have to be coordinated for smooth and collision-free motions

Time Synchronization of Multi-dimensional Trajectories in Joint Space

- Different joints can have **different maximum velocities/ accelerations**, different desired displacements
- **All joints share same motion duration**
- Time synchronization ensures **coordinated, simultaneous actions**
- Common duration determined by **joint with longest motion duration**
- Energy conserved using **minimal necessary acceleration and velocity**

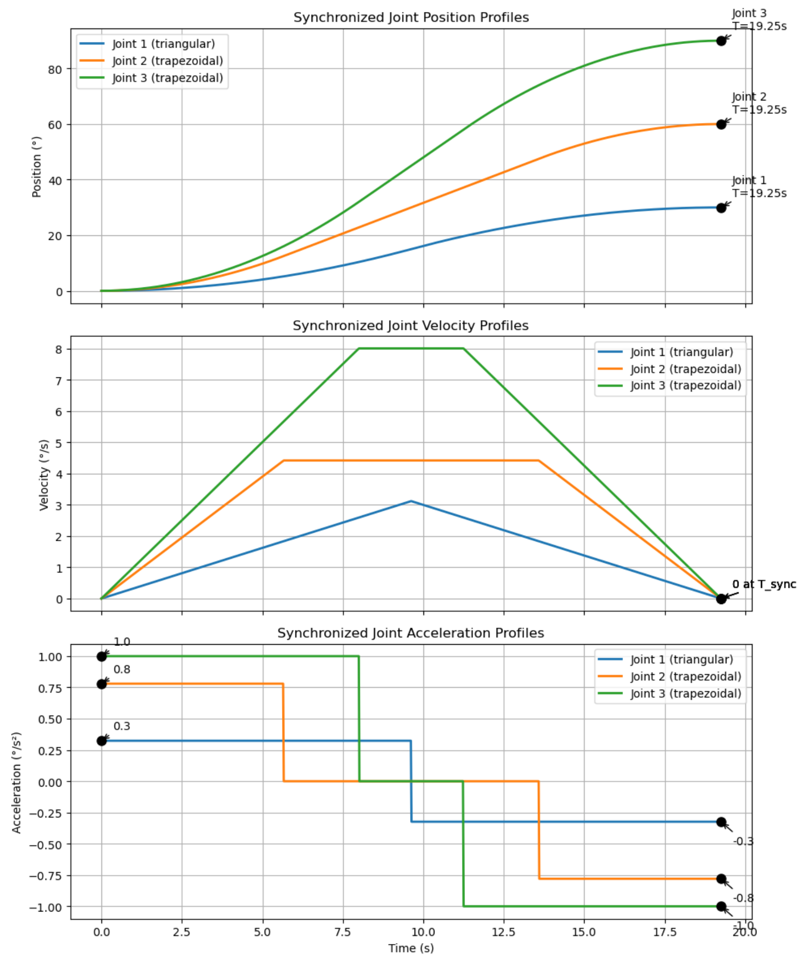
Unsynchronized Trajectories

- Assume robot arm with 3 joints J1, J2, J3 and individual desired changes and maximum velocities
  - J1: $q1_0 = 0, q1_f = 30°, v1_{max} = 6deg/s$
  - J2: $q2_0 = 0, q2_f = 60°, v2_{max} = 5deg/s$
  - J3: $q3_0 = 0, q3_f = 90°, v3_{max} = 8deg/s$
- With unsynchronized motion, all joints reach maximum acceleration $a1_{max} = a2_{max} = a3_{max} = 1\text{deg/s}^2$
- J1 reaches final configuration quickest
- J3 reaches slowest



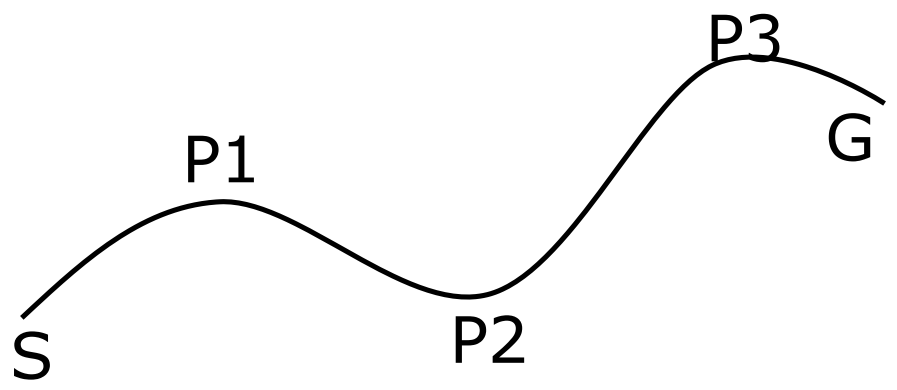
Time Synchronization of Trajectories

- Compute longest trajectory duration T_{sync}
- Calculate stretching factor $\tau_i = t_i / T_{sync}$ where t_i is the unsynchronized time for joint *i*



Multi-Point Trajectory Generation

- Typically motion paths consists of not just S and G, but intermediate waypoints P1, P2, …
- Stopping at each point causes time inefficiency
- Real tasks need smooth, uninterrupted motion
- No need to zero velocity at intermediate points



Multi-Point Trajectory Generation

- Use polynomials to smoothly connect waypoints
- Piecewise cubic trajectory between waypoints

$$q_i(t) = a_i(t - t_i)^3 + b_i(t - t_i)^2 + c_i(t - t_i) + d_i$$

- 4 unknown coefficients per segment: a_i, b_i, c_i, d_i
- Solve for coefficients using boundary conditions
- Match position at each segment boundary

$$q_i(t_i) = q_i, \quad q_i(t_{i+1}) = q_{i+1}$$

Summary

- Definition of basic concepts for motion planning: path, trajectory, configuration space, task space
- Collision detection
- Sampling-based planning considering constraints
- Trajectory planning for generating time-parameterized paths

Literature Trajectory Generation

- Trajectory Planning for Automatic Machines and Robots
  Luigi Biagiotti , Claudio Melchiorri, Springer, 2008