Prof. Dr. Maren Bennewitz
Dr. Marcell Missura
M.Sc. Padmaja Kulkarni

Bonn, 18 June 2017

Humanoid Robots

Exercise Sheet 9 - Reachability maps and bipedal walking

Exercise 16 (10 points)

In this exercise, we will compute reachability maps and inverse reachability maps for the robot arm with three links from exercise 15:

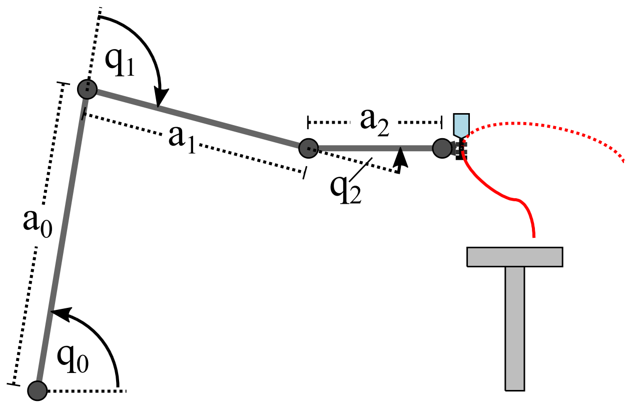

Figure 1: Robot arm with three joints

Reachability map

Compute the reachability map of the endeffector with respect to the robot's base according to the following steps:

a) Implement the method `sampleConfiguration` for choosing a random configuration of the robot's joints (q_0, q_1, q_2) and return the configuration as a vector. The joint angles should be sampled uniformly within the following bounds:

joint	min	max
q_0	0	$+\frac{\pi}{2}$
q_1	$-\pi$	$+\pi$
q_2	$-\pi$	$+\pi$

b) Compute the following measurement of manipulability for a given joint configuration in `computeManipulability()`:

$$\text{score} := 1 - \frac{1}{4\pi}\left(|4q_0 - \pi| + |q_1| + |q_2| + |e_\theta|\right) \tag{1}$$

(This manipulatibility measurement favors configurations where q_0 is near 45 degrees and penalizes configurations with pointed angles between links.)

c) Implement the method `computeRM` that computes the reachability map of the endeffector by iterating the following steps:

- Sample a joint configuration.
- Compute the end effector pose with the `forwardKinematics` method.
- Check that the end effector is above the ground (i.e., $e_y > 0$). If the end effector collides with the ground, then sample again. (You don't have to perform other checks such as self-collisions, object collisions, etc. in this exercise.)
- Compute the manipulability score for the configuration.
- Add the configuration to the reachability map with the `addToRM` method.

With `rosrun irm plot-rm.gp` or in the Wiki you can inspect the resulting reachability map. The robot's base is located at the origin and the colors represent the manipulability of the possible end effector poses.

Inverse reachability map

For the sake of simplicity, we assume that the robot arm is mounted on a wheeled base so that we do not have to deal with stance feet and swing feet.

The inverse reachability map indicates suitable positions where the base of the robot has to be located so that the gripper can reach the desired object. Using this map, the robot can first drive to a suitable place near the table and then grasp the drink.

d) Implement the method `computeIRM`. The voxels of the reachability map are given to the method as an argument. For each configuration stored in the reachability map, add an entry to the inverse reachability map as follows:

- Compute the end effector pose from the configuration's joint angles with the `forwardKinematics` method.
- Convert the end effector pose to a homogeneous transformation matrix, find the inverse, and convert the inverted transformation matrix back to the pose of the robot's base (b_x, b_y, b_θ). The result is the pose of the base expressed in the coordinate system of the gripper.
- Add the base pose together with the joint angles and the manipulability score to the inverse reachability map by calling the `addToIRM` method.

With `rosrun irm plot-irm.gp` or in the Wiki you can inspect the resulting inverse reachability map. The robot's gripper is located at the origin and the colors represent the possible base poses that are suitable for reaching the object in the coordinate frame of the gripper.

Exercise 17 (10 points)

Solve the following questions. Add a new file *src/18_bipedal_walking/bipedal_walking.pdf*, which answers these questions.

a) Formulate the homogeneous transformation matrix of a motion that translates a rigid body by 1.5 meters along the x-axis, 30 cm along the y-axis, and then rotates the body by pi/4.

b) Given the kinematic model as on Slide 19 with l the length of the thigh and the shank, q_1 the angle of the thigh with respect to the trunk, q_2 the angle of the shank with respect to the thigh, q_3 the angle of the foot with respect to the shank, and $v_3 = (v_{3x}, v_{3y})$ the vector pointing from the ankle joint to the toe, construct the homogeneous transformation chain in the form $v_1 = T_1(q_1)\,T_2(q_2)\,T_3(q_3)\,v_3$ that computes v_1, the location of the toe in the frame of the hip joint, and combine the transformation into one matrix.

c) If $l = 0.5$ and $v_3 = (0.1, 0)$, what are the coordinates of the toe if the joint angles are $q_1 = pi/4$, $q_2 = pi/2$, $q_3 = pi/4$?

d) With $l = 0.5$, compute the joint angles q_1, q_2, q_3 that place the toe at the location $v_1 = (0.2$, -0.6) with a foot angle theta = pi/8.

e) What are the abstract interface parameters that achieve exactly the same pose in the question *d*? Note that the leg extension parameter is in the [0,1] range where 0 means the leg is fully retracted and 1 means the leg is fully stretched.

Exercise 18 (10 points)

Using your favorite programming language, program an inverted pendulum simulation in the following four ways: Add a new file $src/18\_bipedal\_walking/bipedal\_walking. < extention >$, to solve these questions. Generate data points after every **10 milliseconds** to plot the graph. Add the graphs in the $src/18\_bipedal\_walking/ < method\_name > .png$

a) Euler integration of the original nonlinear equation with a time step of 0.000001s. This will be taken as ground truth.

b) Euler integration of the original nonlinear equation with a time step of 0.1s.

c) A prediction function based on the linearization around zero.

d) A prediction function based on local linearization.

In each of these simulations, set C = 10 and start the pendulum at angle theta = 0.2 and angular velocity dot theta = 0. Simulate 3 seconds into the future and log the time and the computed angle theta to a file for each of the simulations. Plot the resulting data with gnuplot or a plotter of your choice. Please limit the y-axis to the range [0-7]. Write a brief paragraph answering the following question. Put the plot in the pdf file mentioned above.

a) What observation can you make in the [0-1] second range?

b) What observation can you make in the [1-3] second range?

c) Now change the initial angle to theta = 1.2 and repeat the experiment. Produce a second plot. In what way did the outcome change? Insert the plot in the pdf.

d) Still using C = 10, assume a 2D linear inverted pendulum model with an initial state of (x=-1.0, dot x = 1.0; y = 1.0, dot = -1.0). At what time is the pendulum going to reach a lateral position of y = 2.0?

e) What state will the pendulum have in the sagittal direction at that time?

f) What is wrong with this state?

g) What velocity does the pendulum need to have in order to cross the foot with a pass-through velocity of 1.0?

Exercise 19 (5 points)

Take the nonlinear equation of motion of the pole-cart model from slide 44 and linearize it using a first order Taylor series.
Answer this question in the same file *src/18_bipedal_walking/bipedal_walking.pdf*. Hand solved solution in the image form inserted in the same pdf is also acceptable.

Deadline: 30 June 2017, 11:59 am