

## Humanoid Robots

### Exercise Sheet 3 - Odometry Calibration

#### Exercise 4 (20 points)

You install a surveillance camera on a flag pole in front of the building. The camera's data sheet specifies the following parameters:

principal point	$\begin{pmatrix} x_H \\ y_H \end{pmatrix} = \begin{pmatrix} 400 \\ 300 \end{pmatrix}$
camera constant	$c = 550$
scale difference	$m = 0.0025$

Let the origin of the world coordinate system be at the bottom of the flag pole. The projection center of the camera is located  $Z_0 = 10$  m above the ground and  $X_0 = 40$  cm in front of the flag pole in  $X$  direction.

The camera can be rotated vertically in the  $(Z, X)$  plane around its projection center. Let  $\alpha$  be the current rotation angle in radians. The rotation matrix is then

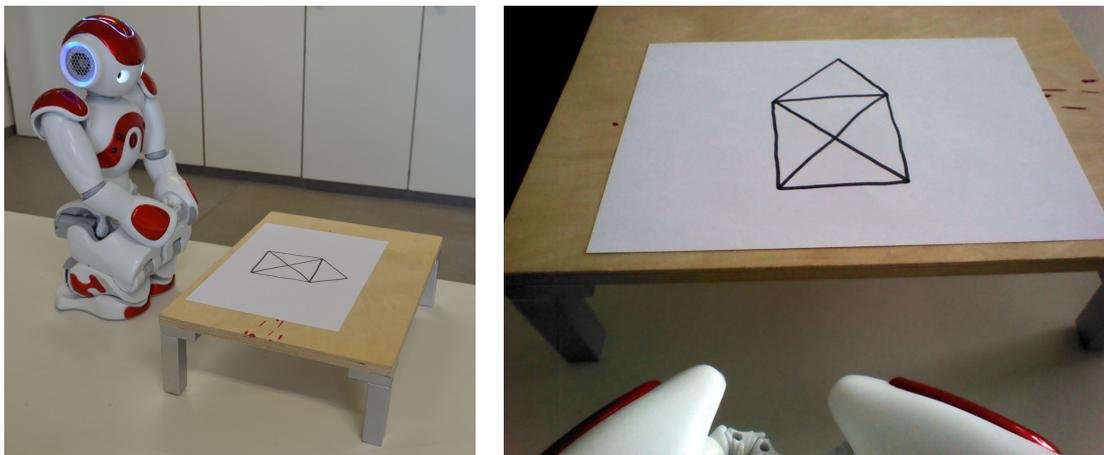
$$R = R_2(\alpha) = \begin{pmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{pmatrix}.$$

- a) Implement the function `euclideanToHomogeneous` that converts Euclidean coordinates in 3D to homogeneous coordinates.
- b) Implement the function `homogeneousToEuclidean` that converts the homogeneous coordinates back to Euclidean coordinates. You may assume that the last component of the homogeneous coordinates is  $\neq 0$ .
- c) Implement the function `setCameraParameters` that returns a data structure that contains the camera parameters.
- d) Implement the function `calibrationMatrix` that returns the calibration matrix  $K$  for the camera parameters given as an argument.
- e) Implement the function `projectionMatrix` that returns the projection matrix  $P$  given the calibration matrix and the rotation angle  $\alpha$ .
- f) Implement the function `projectPoint` that projects a point in 3D coordinates to image coordinates given the projection matrix  $P$ .

## Reprojection from image to 3D

In the lecture, we discussed how to project a given point in 3D space onto a camera image. Many robotics applications require the reverse: The robot detects a feature in the camera image, for example an object that the robot should grasp. It then has to compute the corresponding point in 3D space so that it can move its hand there to grasp the object. Unfortunately, projecting from 3D to the 2D image plane removes one dimension, so reconstructing the 3D point is only possible if an additional piece of information is available, for example the size of the object or a second image from a slightly different perspective.

In the following example, the Nao robot observes a drawing on a table and it is supposed to compute the 3D coordinates of the corners of the house in the drawing. The robot knows that the height of the table is  $h$ , so all points of the drawing will have  $Z = h$  in world coordinates. By exploiting this piece of information, it is possible to reconstruct the full 3D world coordinates of any point on the drawing.



(a) Scene

(b) Robot's view

Figure 1: Nao robot playing a board game.

- g) Implement the function `reprojectTo3D`. Given an input point in image coordinates, the camera parameters, and the table height in meters, the method should calculate and return the corresponding 3D point. *Hint*: Start with the camera equation

$${}^s\mathbf{x} = \mathbf{K} \cdot \mathbf{R} \cdot [\mathbf{I}_3 \mid -\mathbf{X}_O] \cdot \mathbf{X} \quad (1)$$

$$\Leftrightarrow \begin{pmatrix} {}^s x \\ {}^s y \\ 1 \end{pmatrix} = \mathbf{K} \cdot \mathbf{R} \cdot \begin{pmatrix} 1 & 0 & 0 & -X_O \\ 0 & 1 & 0 & -Y_O \\ 0 & 0 & 1 & -Z_O \end{pmatrix} \cdot \begin{pmatrix} U \\ V \\ W \\ T \end{pmatrix}. \quad (2)$$

There are three equations for the four unknown components  $U, V, W, T$  of the 3D point  $\mathbf{X}$  in homogeneous coordinates, so the system is underdetermined. Additionally, the table height is given, which yields the equation

$$\frac{W}{T} = h. \quad (3)$$

Solve Eq. 3 for  $T$ , substitute  $T$  in Eq. 2 and rewrite the equation in the form

$$\begin{pmatrix} {}^s x \\ {}^s y \\ 1 \end{pmatrix} = \mathbf{K} \cdot \mathbf{R} \cdot \mathbf{A} \cdot \begin{pmatrix} U \\ V \\ W \end{pmatrix} \quad (4)$$

where  $\mathbf{A}$  is a  $3 \times 3$  matrix. Solve the linear system, compute  $T$  and convert back from homogeneous to Euclidean coordinates.

**Deadline: 2 May 2019, 11:59 am**