

## Humanoid Robots

### Exercise Sheet 7 - Anytime Repairing A\* and Zero Moment Point

#### Exercise 13 (10 points)

Extend the grid-based A\* planner from exercise 11 to use Anytime Repairing A\* (ARA\*).

Exercise steps:

- Implement the ARA\* heuristic that inflates the Euclidean heuristic by a factor  $w$  in `ARAStarHeuristic::heuristic`.
- “Anytime” algorithms continuously try to improve their solution until a time limit is met. Hence, the planning algorithm must be able to abort the search for a plan. Copy your code from the `PathPlanning::planPath` of exercise 11 to `ARAStarPlanning::planPath` and modify it so that it aborts searching when the time frame is exceeded. The method should then return an empty path.
- Implement the `ARAStarPlanning::runARA` method that runs the path planner iteratively and decreases  $w$  in each iteration until either the optimal path is found, or the time limit has exceeded.

The Gnuplot script in the `scripts` directory and the animation in the Wiki show three plots: The figure on the left shows the path in red and the expanded nodes for each  $w$  in light blue. The top right figure shows how the path length of the returned path changes when  $w$  decreases. The bottom right figure shows how the planning time for each iteration increases with decreasing  $w$ .

### Exercise 14 (10 points)

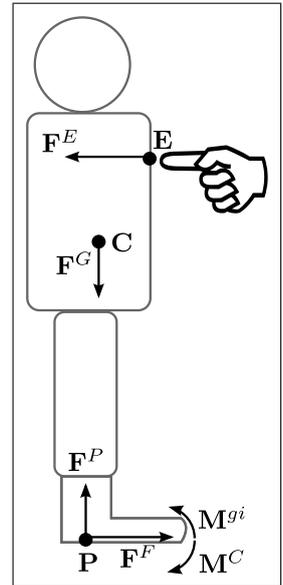
A robot is standing upright. Now a human exerts a force on the robot's chest, pushing the robot backwards (see figure on the right). How strong can the human push the robot before it starts tipping over backwards?

In this exercise, you will calculate the involved forces and torques and use the Zero Moment Point (ZMP) to estimate whether the robot will fall.

Implement the methods in `src/ZMP.cpp` with the help of the explanations below and the symbol table Table 1.

In our scenario, four forces act on the robot:

- The gravitational force  $\mathbf{F}^G$  acts on the center of mass  $\mathbf{C}$  and forces the robot to the ground.
- The user exerts an external force  $\mathbf{F}^E$  acting on point  $\mathbf{E}$  located on the robot's chest.
- The ground reacts with a pressure force that prevents the robot from sinking into the ground. The pressure force acts on the whole contact area, but it can be substituted by a single force  $\mathbf{F}^P$  acting at the center of pressure  $\mathbf{P}$ .
- The friction force prevents the robot from sliding backwards and it acts on the whole contact area. It can again be substituted by a single force  $\mathbf{F}^F$  acting on the center of pressure  $\mathbf{P}$ . We assume that the friction is high enough so that the robot never slides.



We assume that the robot is standing still, so according to Newton's first law the forces sum up to zero:

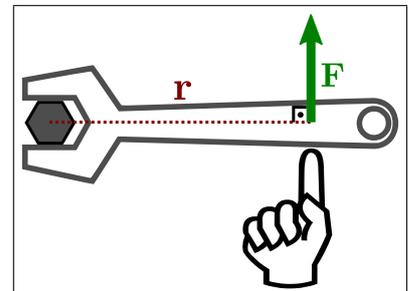
$$\underbrace{\mathbf{F}^G + \mathbf{F}^E}_{=: \mathbf{F}^{gi}} + \underbrace{\mathbf{F}^P + \mathbf{F}^F}_{=: \mathbf{F}^C} = \mathbf{0} \quad (1)$$

- Calculate  $\mathbf{F}^{gi}$  in the method `giForces()`.
- Calculate  $\mathbf{F}^C$  in the method `contactForce()` using  $\mathbf{F}^{gi}$  and Eq. 1.

Forces acting on a rigid body induce *angular moments*, also called *torques* (not to be confused with angular momentum). In the figure on the right, for example, a force  $\mathbf{F}$  acting on a wrench induces a torque

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} \quad (2)$$

on the screw, where  $\mathbf{r}$  is the lever arm,  $\times$  is the cross product, and  $\mathbf{F}$  is the force acting on the wrench.



The forces acting on the robot induce different angular moments in each point  $\mathbf{X}$  of the robot. For example, the gravity force  $\mathbf{F}^G$  induces a moment  $\mathbf{M}^G = (\mathbf{C} - \mathbf{X}) \times \mathbf{F}^G$  as the gravity acts on the center of mass.

According to Newton's first law, the moments in every point  $\mathbf{X}$  also sum up to zero if the robot is standing still:

$$\underbrace{\mathbf{M}^G(\mathbf{X}) + \mathbf{M}^E(\mathbf{X})}_{=: \mathbf{M}^{gi}(\mathbf{X})} + \underbrace{\mathbf{M}^P(\mathbf{X}) + \mathbf{M}^F(\mathbf{X})}_{=: \mathbf{M}^C(\mathbf{X})} = \mathbf{0} \quad (3)$$

c) Calculate  $\mathbf{M}^{gi}$  in `giMoments()`.

We would like to determine whether the robot tips over or not, so we are only interested in the horizontal components of the moment  $\mathbf{n} \times \mathbf{M}$ .

The zero moment point is defined as the point  $\mathbf{Z}$  where the horizontal components of both the gravity-inertial moments  $\mathbf{M}^{gi}$  and the contact moments  $\mathbf{M}^C$  are zero:

$$\mathbf{n} \times \mathbf{M}^{gi}(\mathbf{Z}) = \mathbf{n} \times \mathbf{M}^C(\mathbf{Z}) = 0 \quad (4)$$

Given an arbitrary origin  $\mathbf{O}$  of the ground plane coordinate system, the point  $\mathbf{Z}$  that fullfills Eq. 4 can be computed using

$$(\mathbf{Z} - \mathbf{O}) = \frac{\mathbf{n} \times \mathbf{M}^{gi}(\mathbf{O})}{\mathbf{F}^{gi} \cdot \mathbf{n}}. \quad (5)$$

This equation results from the general torque formula Eq. 2 solved for  $\mathbf{r}$  and projected onto the contact plane.

d) Calculate the zero moment point in `zeroMomentPoint()` using Eq. 5.

As long as the zero-moment point is within the support polygon, it coincides with the center of pressure  $\mathbf{P}$  where the ground reaction force acts on the robot. When the (virtual) zero-moment point leaves the support polygon, the center of pressure cannot follow, as pressure can only occur where the robot touches the ground.

e) Compute the ground reaction moment  $\mathbf{M}^C(\mathbf{X})$  relative to point  $\mathbf{X}$  in the method `contactMoment()`. The location of the center of pressure  $\mathbf{P}$  and the point  $\mathbf{X}$  are given as arguments to the method.

f) Compute the resulting angular moment at the center of mass  $\mathbf{M}^{gi}(\mathbf{C}) + \mathbf{M}^C(\mathbf{C})$  in `resultingMomentAtCoM()`.

According to Newton's first law, the robot will stay upright if the angular moment at the center of mass is zero and it will tip over if the angular moment is non-zero.

The Gnuplot script in `scripts/plot.gp` and the animation in the Wiki show the distribution of moments and pressure as the external force increases.

**Deadline: 6 July 2017, 11:59 am**

Table 1: Symbols used in the ZMP exercise

Symbol	Unit	Name / description	C++
$m$	kg	mass	<code>m</code>
$\mathbf{g}$	m/s <sup>2</sup>	gravitational acceleration	<code>g</code>
$\mathbf{n}$	1	normal vector of the contact plane	<code>n</code>
$\mathbf{C}$	m	center of mass	<code>C</code>
$\mathbf{P}$	m	center of pressure	<code>CoP</code>
$\mathbf{E}$	m	point where the external force acts	<code>E</code>
$\mathbf{O}$	m	origin of the contact plane coordinate system	<code>O</code>
$\mathbf{Z}$	m	zero-moment point	<code>zmp</code>
$\mathbf{F}^G$	N	gravitational force	
$\mathbf{F}^E$	N	external force	<code>F_E</code>
$\mathbf{F}^P$	N	ground pressure force	
$\mathbf{F}^F$	N	friction force	
$\mathbf{F}^{gi} = \mathbf{F}^E + \mathbf{F}^P$	N	gravity-inertial force	<code>giForce()</code>
$\mathbf{F}^C = \mathbf{F}^P + \mathbf{F}^N$	N	ground reaction force, contact force	<code>contactForce()</code>
$\mathbf{M}^{\{G,E,P,F\}}(\mathbf{X})$	Nm	angular moment, torque with respect to point $\mathbf{X}$ (superscripts same as for $\mathbf{F}$ )	
$\mathbf{M}^{gi} = \mathbf{M}^E + \mathbf{M}^P$	Nm	angular moment induced by gravity-inertial forces with respect to point $\mathbf{X}$	<code>giMoments()</code>
$\mathbf{M}^C = \mathbf{M}^P + \mathbf{M}^N$	Nm	angular moment induced by contact force with respect to point $\mathbf{X}$	<code>contactMoments()</code>